IMPROVEMENT OF CONTROL VALVE PERFORMANCE

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by

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CONTENTS

ABSTRACT

This thesis is concerned with improving the performance of the control valve erected on a One Foot Wind Tunnel in National Aeronautical Laboratory Bangalore. Improved performance will increase the useful time available for testing the aero models for a definite quantity of air blown from the receiver. The first two chapters deal with the problem of locating the cause for the malperformance of the existing system and obtaining a close mathematical model from the given data obtained by experiment and furnished by the laboratory. A compensator has been designed to modify the mathematical model to yield improved performance. The modified system mathematical model after compensation has been simulated on an Analog Computer. The improved performance has been demonstrated both by Analog Simulation and digital verification of the Analog model. The results of both existing and compensated system response have been tabulated for comparison.

CHAPTER 1

INTRODUCTION

Technical problems of immediate concern frequently relate only to a subpart of an elaborately organised system. Such is the case here. Consequently an appreciation of the detailed study can only be achieved after having a cursory glance at the entire system. It is the aim to provide some background in this respect.

It is of considerable importance in the aircraft and missile industry now-a-days to obtain information about aircraft stability and aerodynamic parameters prevailing at mach numbers ranging from .5 to 5 and higher. Several of the leading aircraft companies have built wind tunnels to accomplish this purpose. The wind tunnel is often of the blow-down type. The compressors pump air to a specified pressure in huge storage tanks. When a test is to be performed the stored air is bled through a valve into a settling chamber where air flow at high mach number is produced. In such tests, keeping the mach number constant is of the utmost importance. Unfortunately, as the air is bled from the storage tanks, its pressure drops and so too does the mach number in the test section unless the valve is adjusted. The need for a control system therefore presents itself. The nature of the device depends upon the means

available for monitoring the quantity it is desired to control, which in this case is the mach number. Since the mach number can be identified in terms of the pressure, it turns out that to keep the mach number fixed requires keeping the settling chamber pressure constant. Hence the use of a pressure activated servo system suggests itself. It is interesting to note that the mach number is now the indirectly controlled variable while the pressure is the directly controlled variable.

A schematic diagram of the wind tunnel without automatic pressure control is shown in Fig. 1.1. The block diagram of the control system including the components required is shown in Fig. 1.2.

The air is stored in reservoir tanks at a fixed pressure depending upon the desired mach number to be achieved. The tanks discharge into a fixed diameter manifold which in turn leads to a rotating plug control valve. The discharge side of the valve is connected to the settling chamber by means of a duct. The fixed mach number is obtained by blowing air through a nozzle from the settling chamber. The shape of the adjustable diffuser controls the particular mach number for a constant settling chamber pressure.

The pressure transducer is a device which converts pressure to a corresponding voltage, which in turn is compared with the reference input voltage. The difference constitutes the actuating signal. The controller refers to an amplifying stage plus electrical networks which modify the input signal.

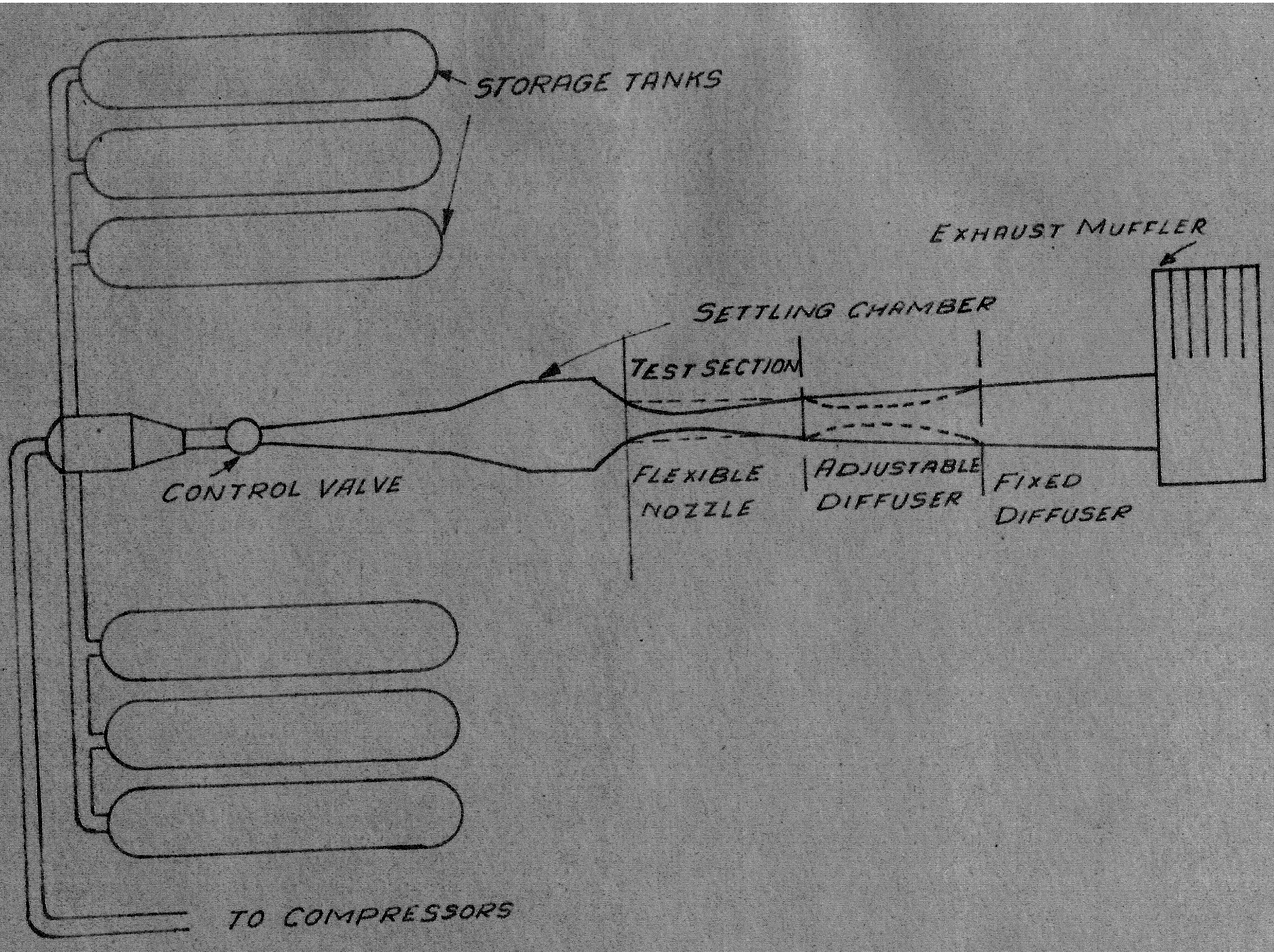


FIG. 1.1 A HIGH-SPEED WIND TUNNEL.

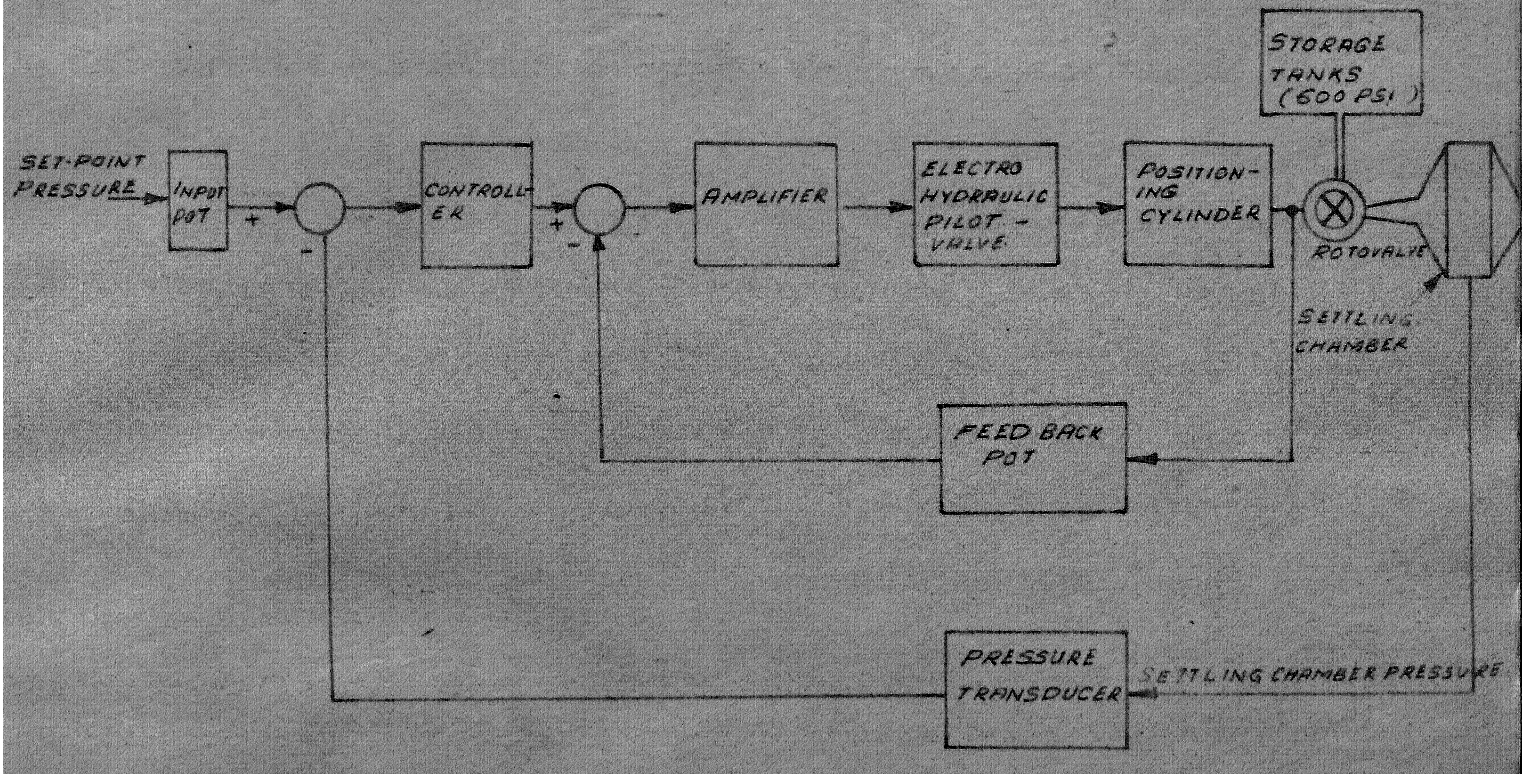


FIG. 1-2 BLOCK DIAGRAM OF A WIND-TUNNEL PRESSURE CONTROL SYSTEM.

The electro-hydraulic valve is an electrically operated two stage hydraulic amplifier. It consists of a coil-magnet motor, low pressure and a high pressure pilot valves. The output motion of the latter drives the positioning cylinder which in turn positions the roto valve. For improved performance a position feedback is put around the electrohydraulic valve. The controller, the electrohydraulic valve circuit and the roto valve represent the feedback control system. The error detection is achieved through the input potentiometer and the feedback transducer.

Assume that a wind tunnel test is to be performed at mach 5. This corresponds to a settling chamber pressure of 250 p.s.i. Operation is started by putting the input potentiometer at a set point pressure of 250 p.s.i. An actuating signal immediately appears at the controller which in turn causes the electrohydraulic pilot valve and its positioning cylinder to open the roto valve and thereby build up pressure in the chamber. As the roto valve is opened the input to the pilot volve amplifier decreases because of the position feedback voltage. Moreover, as pressure builds up, the actuating signal to the controller decreases. When the settling chamber pressure has reached the commanded 250 p.s.i., the actuating signal will be zero and no further movement of the roto volve plug takes place. The time needed to accomplish this is relatively small and the attendant decrease in tank pressure is also very small. Consequently an air flow of mach 5 is established in the settling chamber. However as time passes more and more air is bled from the storage tanks,

the storage pressure will decrease unlessa a further opening of the control valve is introduced, the settling chamber pressure will drop also. Of course additional roto valve opening occurs because of the pressure feedback. Specifically, as the settling chamber pressure decreases the corresponding feedback voltage drops. In as much as the reference input voltage is fixed an actuating signal appears at the controller, which causes the pilot valve to reposition the roto valve. This inturn maintains the desired chamber pressure. It should be clear that this corrective action will prevail as long as there is any tendency for the chamber pressure to drop and provided that the roto volve is not at its limit position i.e. not fully open.

The wind tunnel erected at the National Aeronautical Laboratory, Bangalore is shown in Fig. 1.3. In principle the operation is that previously described.

The problem is concerned with improving the performance of the control valve erected on the ONE FOOT WIND TUNNEL described in Fig. 1.3. Improved performance would increase the time duration over which the aeromodel experiments could be conducted under constant mach number for a definite quantity of the air blown from the storage tank. This will amount to reduced operational cost for a fixed length of test time.

The following observations were made on the operating system:

1. The given system was stable.
2. The system was extremely sluggish and possessed an excessive settling time.

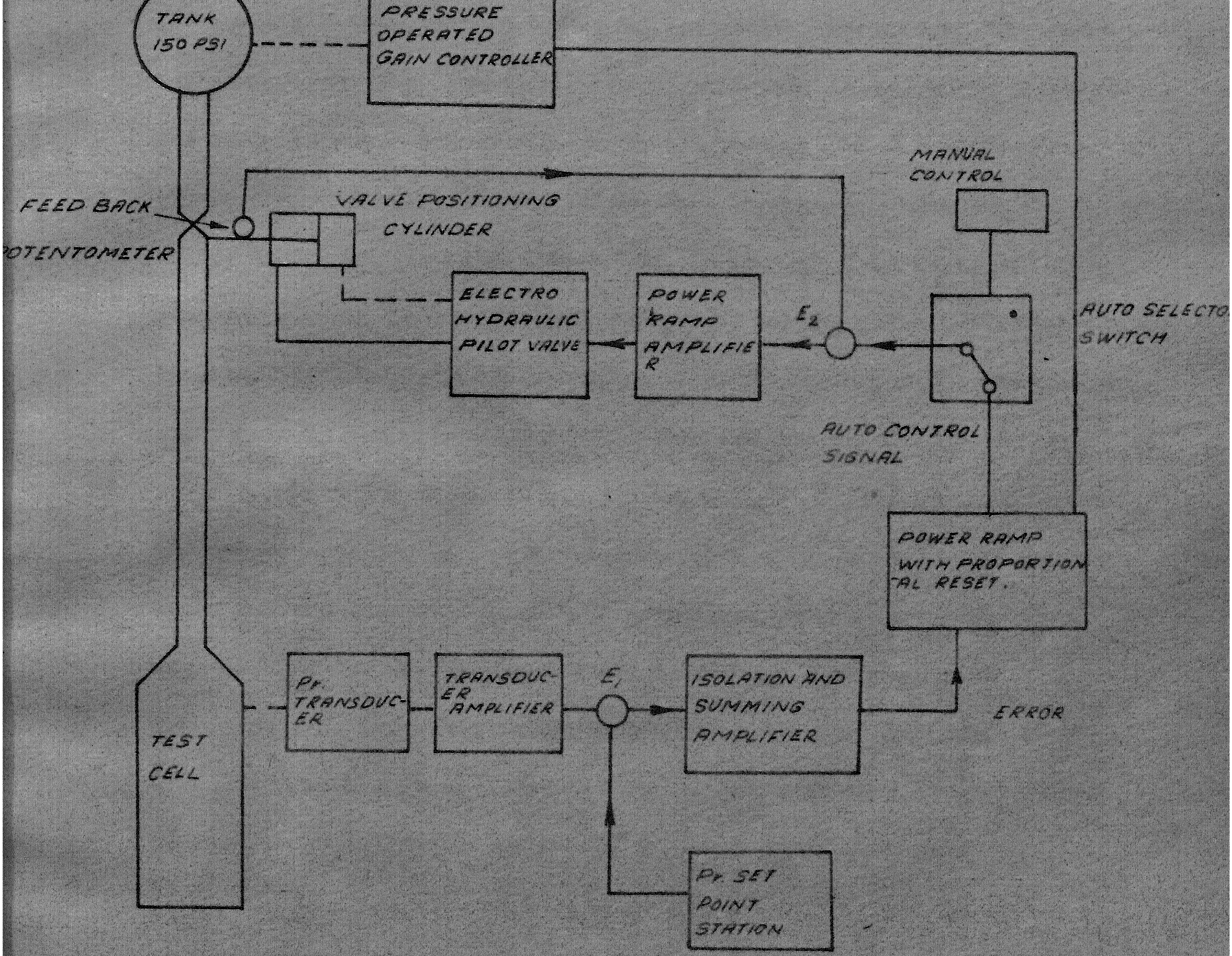


FIG. 1·3 BLOCK DIAGRAM OF THE CONTROL SETUP ERECTED IN NATIONAL-AERONAUTICAL LABORATORY.

E_1 REPRESENTS A SUMMING NETWORT LOCATED IN THE INPUT BOX OF THE ISOLATION AND SUMMING AMPLIFIER.

E_2 REPRESENTS ERROR SIGNAL RESULTING FROM A DIFFERENCE BETWEEN A VALVE POSITION AND THAT CALLED FOR BY THE CONTROL SIGNAL.

3. An increase in gain decreased stability without markedly improving the settling time.

The aim of the work is to analyse the given system so as to reduce the settling time without exceeding the permissible overshoot. The steady state error also should be brought to as low a value as possible. Analytical work is centred around a set of data forming a closed loop Bode diagram. The frequency response of the above system was taken for the closed loop case. The closed loop Bode plot revealed the presence of a resonant rise on the negative db. region. By obtaining the closed loop frequency response plot for the individual dynamic components and comparing it with the closed loop plots it was inferred that the structural resonance is due to the column of liquid resonating on the link pipe connecting the hydraulic ram to the hydraulic accumulator. At low values of gain its effect on the system was found to be negligible, as it occurs well below zero db line. On the otherhand it was a dominant factor when the gain was increased, thereby causing disturbance on the system. After locating the trouble spot in the system two lines of attack for the solution became evident.

1. Modifying the existing system in order to reduce the resonant rise i.e. the physical parameters which have bearing on the resonance could be altered in the system in operation to overcome the illeffect.

2. Instead of changing the parameters of the existing system a compensator could be added to the physical system to alter its performance.

The second method was preferred as the former would probably call for a major and costly overhaul of the entire physical plant.

CHAPTER II

MATHEMATICAL MODEL OF THE SYSTEM

In order to improve the performance of a system it is essential to obtain the mathematical model. The tigiven closed loop system was subjected to sinusoidal input in the laboratory and the output response was noted for different frequencies. From the observed data a Bode plot was drawn for analysis. Fig. 2.1. represent the closed loop magnitude and phase shift plots of the given system. The figure indicates two overshoots on the system. From the individual frequency plot of the hydraulic ram shown in Fig. 2.2. the cause of the first overshoot was located. The presence of the small amplitude non linearity shown in Fig. 2.2. has been neglected in obtaining the mathematical model.

The system being linear, for all practical purposes the magnitude plot could be split into subgraphs each representing either a pole, a zero or a quadratic term. The closed loop transfer function could be obtained by knowing the corner frequencies for the individual separated plots. The closed loop equation obtained by the above procedure is of the form

$$C/R\ (S) = \frac{(1 + .08S)\ (1 + .0177S)\ 732 \times 10^3}{(S^2 + 44.8S + 12{,}200)\ (S^2 + 6.28S + 60)} \qquad (1)$$

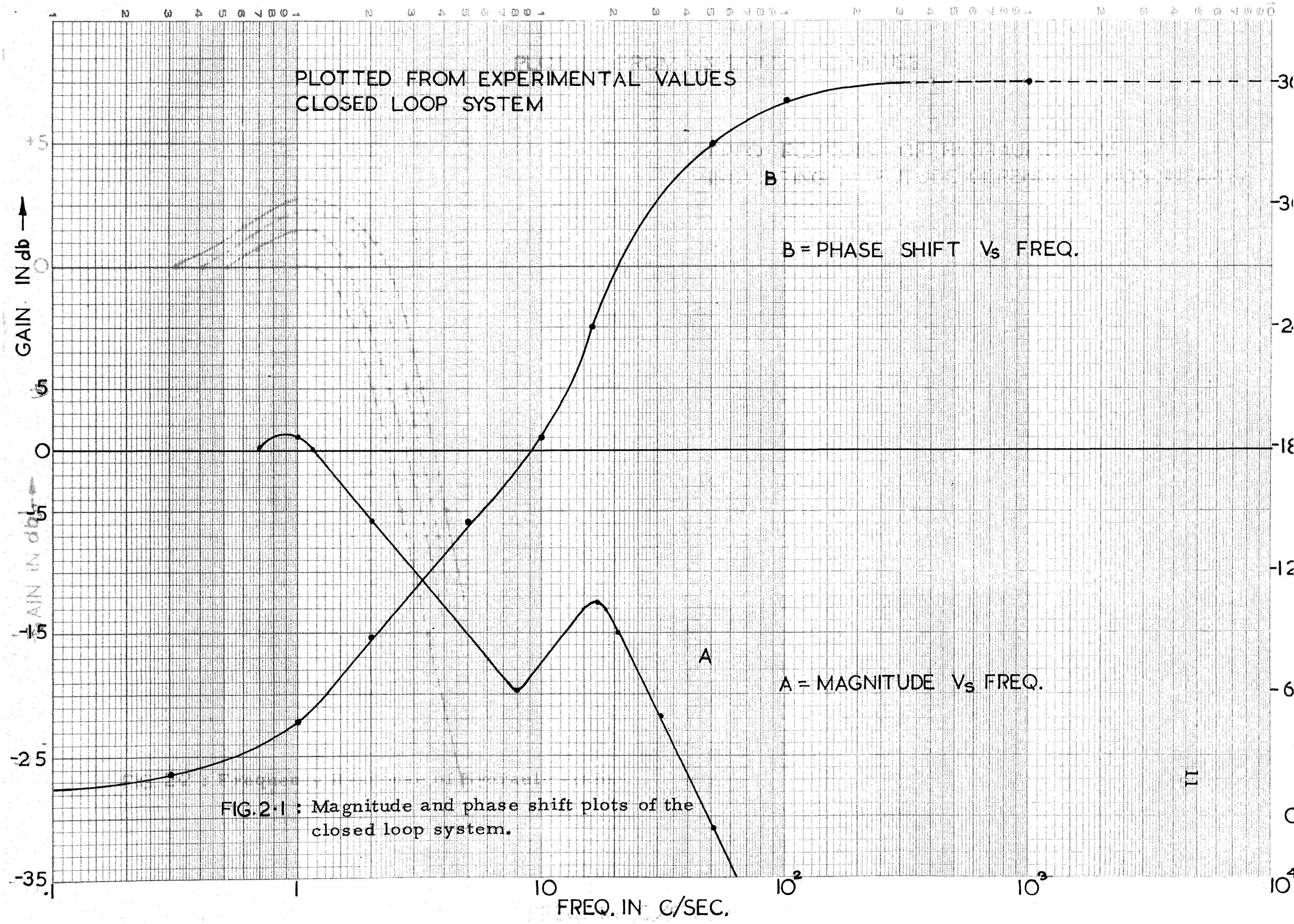


FIG. 2·1 : Magnitude and phase shift plots of the closed loop system.

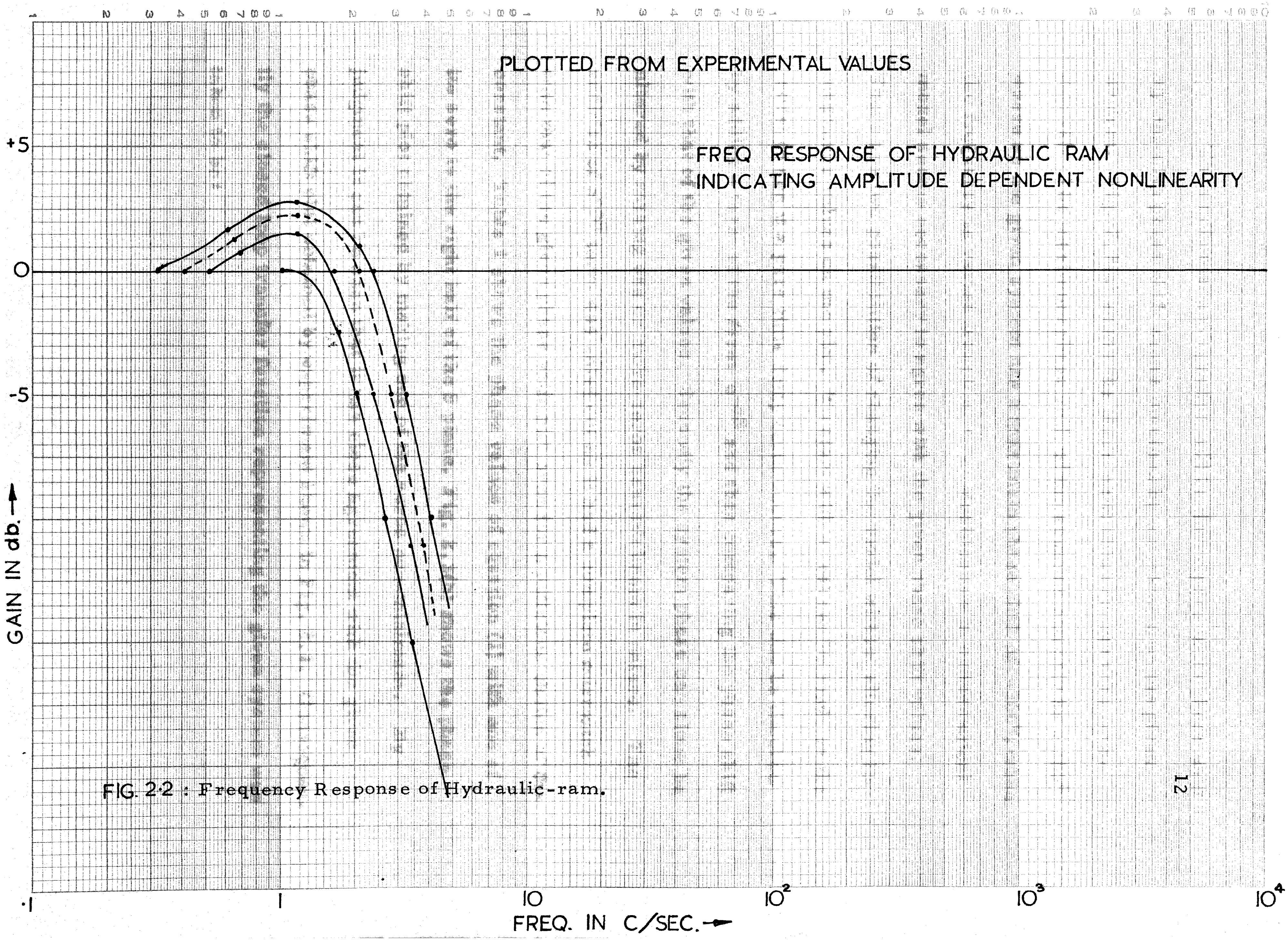


FIG. 2·2 : Frequency Response of Hydraulic-ram.

The mathematical equation will be descriptive if its phase shift agrees with phase shift values obtained by experiment on the given system. Fig. 2.3(a) shows the phase shift obtained from the equation (1). The maximum phase shift of the equation can only be -180° where as the given system has a maximum phase shift of -360°; when the frequency is varied from 0 to ∞. Modification in mathematical model seems necessary to agree with the given phase shift values. There are two possibilities of changing the model without altering its magnitude values on the plot. To start with a test can be made on the model to see if a pure time delay term occurs. This can be checked by adding a term of the form e^{-aS} and then comparing the phase shift with that of the given values. Secondly the system phase can alone be altered by making one or more zeros non-minimum in nature. The addition of e^{-aS} term was ruled out as it had a complete mismatch with that of the given phase shift plot even for a judicious choice of the constant. Table 1 shows the phase values of equation (1) with one of its zero in the right half of the S plane. Fig. 2.3(b) shows the phase shift plot obtained by making one of the zeros non-minimum. By judgment it can be seen that it has close agreement with the given phase shift plot obtained by experiment shown in Fig. 2.1. Consequently the closed loop transfer function representing the given system was taken to be:

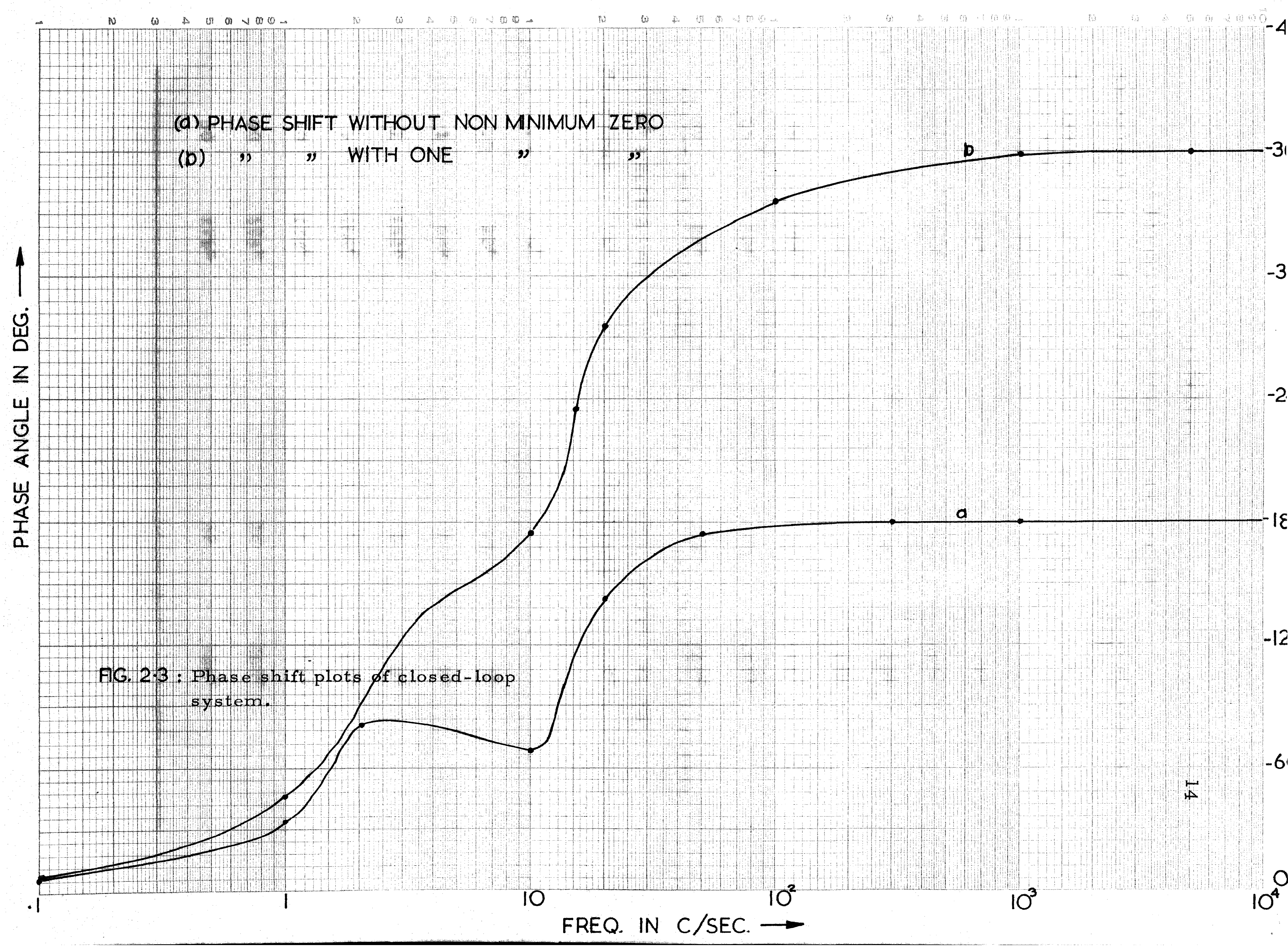


FIG. 2·3 : Phase shift plots of closed-loop system.

TABLE 1

Phase Angle (Closed Loop System)

$(1 + .08S)$		$(1 - .0177S)$	
Frequency (in c/s)	θ°	Frequency (in c/s)	θ°
.1	3°	.1	-
.5	14°	.5	-3°
1	27°	1	-7°
2	45°	2	-13°
5	68°	5	-29°
10	79°	10	-48°
20	84°	20	-65°
50	88°	50	-80°

$S^2 + 6.28\,S + 60$		$S^2 + 44.8S + 12,200$	
Frequency (in c/s)	θ°	Frequency (in c/s)	θ°
.1	4°	.1	-
.5	22°	.5	1°
1	60°	1	2°
2	131°	2	3°
5	168°	5	17°
10	174°	10	31°
20	175°	20	120°
50	176°	50	158°

$$C/R\ (S) = \frac{(1 + .08S)(1 - .0177S)\ 732 \times 10^3}{(S^2 + 44.8S + 12,200)(S^2 + 6.28S + 60)} \qquad (2)$$

It must be clearly remembered that above procedure of obtaining a close fit is a compromise between error and complexity. The system has one zero in the right half of the S plane thereby making it non-minimum in nature. The presence of the non-minimum term is bound to make the system analysis complex as certain Bode theorems and techniques to study the performance are inapplicable.

OPEN-LOOP TRANSFER FUNCTION

Knowing the system has unity feedback one can proceed to obtain the open loop equation in the following fashion. To begin with C/R (S) has the general form shown below:

$$C/R\ (S) = \frac{1 + a_1 S + a_2 S^2}{1 + b_1 S + b_2 S^2 + b_3 S^3 + b_4 S^4} \qquad (3)$$

For a negative unity feedback system

$$C/R\ (S) = \frac{G(S)}{1 + G(S)} = \frac{1}{1 + \left(\frac{1}{G(S)}\right)} \qquad (4)$$

The equation (3) can be brought to the standard form by dividing the numerator into the denominator, thus:

$$
\begin{array}{r|l}
 & 1 \\
\hline
1 + a_1S + a_2S^2 & 1 + b_1S + b_2S^2 + b_3S^3 + b_4S^4 \\
 & 1 + a_1S + a_2S^2 \\
\hline
 & (b_1 - a_1)\,S + (b_2 - a_2)\,S^2 + b_3S^3 + b_4S^4
\end{array}
$$

The transfer function reduces to the form

$$
\frac{1}{1 + \dfrac{S\,(b_1{}' + b_2{}'S + b_3S^2 + b_4S^3)}{(1 + a_1S + a_2S^2)}}
$$

where it is clear that the open and closed loop zeros are identical as they should be.

A comparison with the standard form reveals that

$$
G\,(S) = \frac{(1 + a_1S + a_2S^2)}{S\,(b_1{}' + b_2{}'S + b_3S^2 + b_4S^3)}
$$

The polynomial which in this case is of third order was solved for its roots on the computer. By the above procedure the open loop transfer function obtained turned out to be

$$
G\,(S) = \frac{22.7\,(1 + .08S)\,(1 - .0177S)}{S\,(1 + .418S)[(S^2/13,376) + (48.4S/13,376) + 1]} \tag{5}
$$

where it may be noted that the system is of type 1 with a velocity constant K_v equal to 22.7.

The open loop poles are

$S_0 = 0 \quad S_1 = -2.4$

$S_{2,3} = -24.2 \pm j\ 113.4$

Table 2 shows the phase values pertaining to the open loop equation (5). The Bode plots (Magnitude and phase shift) are shown in Fig. 2.4. To explain the effect of overshoot the magnitude plot has been drawn for two gains $K_v = 22.7$ and $K_v = 120$.

For the given system gain of $K_v = 22.7$ it may be seen that

The phase margin of the system $= 40^\circ$

The gain margin of the system $= 20$ db

The system has an excessive gain margin for the loop gain value of $K_v = 22.7$. In any system excess stability indicates or reveals sluggish response. In this system an increase in open loop gain brings the phase and gain margin to a reasonable value thereby corrects for the excess stability. So the sluggish performance of the control valve could have been taken care of by the increase in open loop gain only. However because of the presence of overshoot in the negative db region such modification in gain alone cannot serve the purpose.

When the system gain has been increased the resonant rise moves over to the active region in the Bode plot. In the open loop Bode plot any overshoot with in $\pm$ 15 db region will have considerable effect on the system performance. Such a system will have poor transient performance because of the overshoot and it was verified to be correct later by Analog simulation.

TABLE 2

Phase Angle (Open Loop System)

(1 + .418S)		(1 + .08S)	
Frequency (in c/s)	θ°	Frequency (in c/s)	θ°
.1	15°	.1	3°
.2	28°	.2	5°
.5	53°	.5	14°
1	69°	1	27°
2	79°	2	45°
5	86°	5	68°
10	87°	10	79°
50	89°	50	88°

(1 - .0177S)		$S^2 + 48.4S + 13{,}376$	
Frequency (in c/s)	θ°	Frequency (in c/s)	θ°
.1	-	.1	-
.5	-3°	.5	-
1	-7°	1	-
2	-13°	2	1°
5	-29°	5	6°
10	-48°	10	17°
20	-65°	20	115°
50	-80°	50	150°

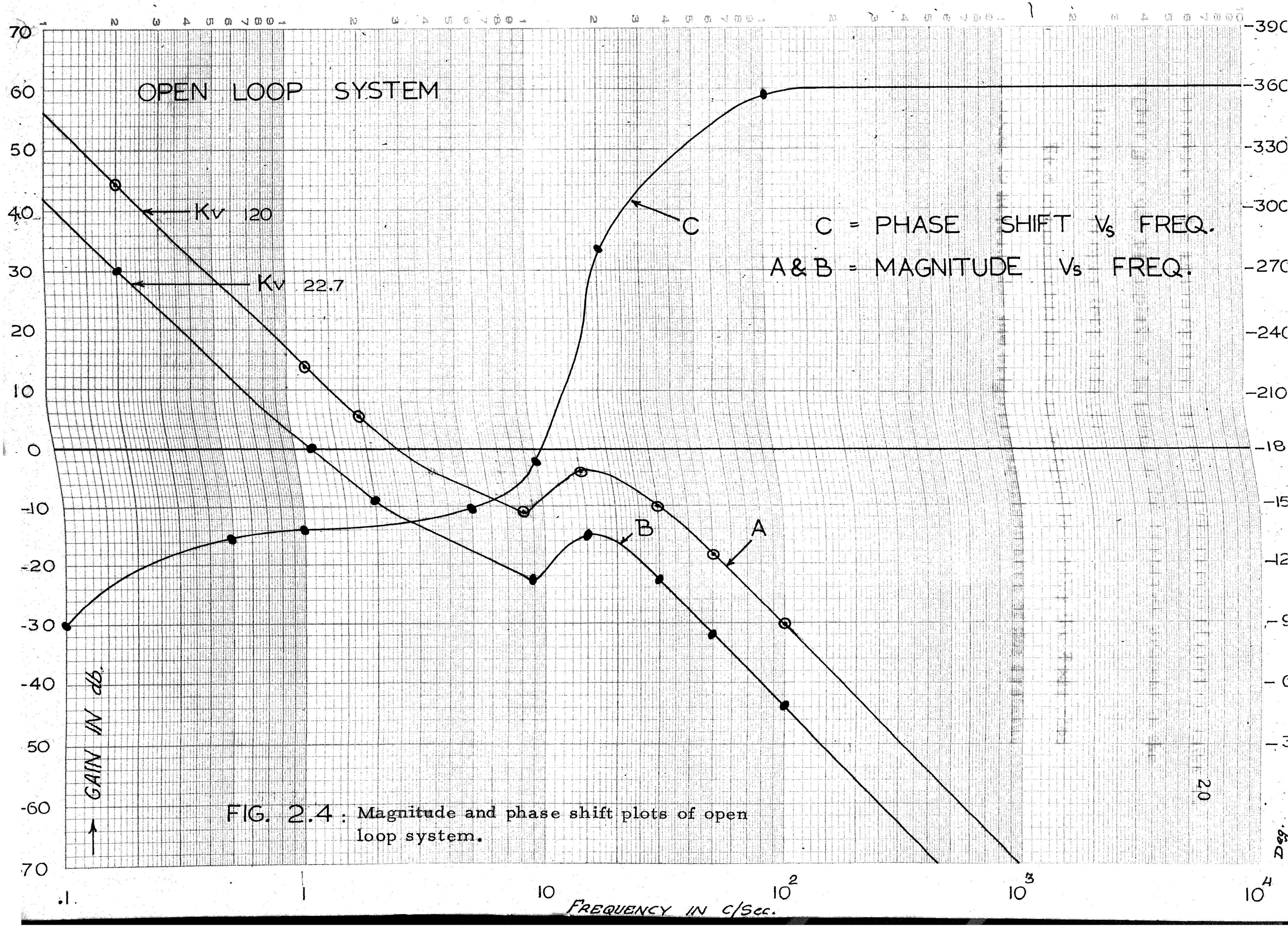


FIG. 2.4 : Magnitude and phase shift plots of open loop system.

Therefore in the above system the poor performance of control valve at higher gain can be explained in terms of the overshoot present in the active region in open loop Bode plot.

So a compensator has to be designed to modify the Bode plot in order to account for the original malperformance of the system.

CHAPTER III

COMPENSATION

The open loop magnitude plot drawn in figure 2.4 reveals that the given system has an excess gain margin for $K_v = 22.7$. The objective is to obtain a reasonable settling time without exceeding permissible overshoot. From the phase shift plot shown in Fig.2.4 it can be seen that the phase margin remains more or less constant for a large increase in the open loop gain. The phase margin of a simple quadratic system has a relationship with δ the damping ratio of the system. Although this relationship is not directly valid for higher order systems the qualitative correlation is generally correct and useful. Since the δ of the given system remains approximately constant (for an increase in the open loop gain) the response could be speeded up without altering the system stability. This has also been verified by analog simulation. The system gain could be increased to a convenient value so that a reasonable gain margin and phase margin would be obtained. The improved performance could have been obtained by increasing the loop gain to a moderate value if there had not been the degrading transient effect due to resonance phenomena. So it is obvious thatt the system is being operated at low gain to avoid

transient instability caused by the resonance phenomena. A change in gain would take care of the desired improvement if the high frequency resonance overshoot at that increased gain value was attenuated without altering the other characteristics.

The problem now reduces to that of designing a realizable compensator to achieve this result. The compensator theoretically designed must be physically realizable. It may be placed in the following locations:

1. In cascade with the forward transfer function
2. In the feedback path. The selection of the location for inserting the compensator is dependent largely on the result desired.

CASCADE COMPENSATION

Generally the cascade compensators are inserted at the low energy point in the forward path so that power dissipation is small. This also requires that the input impedance be high. The use of isolation amplifiers may be necessary to avoid loading. This is particularly true in multiple section networks.

The cascade networks used for compensation are generally called lag lead and lag lead types. The aim is to attenuate the overshoot alone in the Bode plot without altering the plot in the other regions. The phase shift also must not be appreciably altered in the non resonance region. It is clear from the constraints that no cascade compensator will serve the purpose. Since any effort to change the magnitude plot near the resonance region will change the phase shift graph in other regions thereby affecting

the performance indices.

FEEDBACK COMPENSATION

The necessary compensation can be obtained by introducing a compensator in an additional feedback loop as shown in Fig. 3.1. The combined transfer function with feedback element H (S) included is given by

$$G_1 (S) = \frac{G_M (S)}{1+G_M(S)H(S)}$$

$G_M (S)$ is the transfer function of the forward control. H (S) is the transfer function of the feedback element.

The transfer function $G_1 (S)$ may be handled in an approximate fashion as shown below:

$$1 + G_M(S)\ H(S) \rightarrow 1 \qquad 1 \gg G_M (S)\ H(S)$$

$$\text{In this region } G_1 (S) \rightarrow G_M (S)$$

The value of H (S) can be chosen such that the $G_M(S)$ H(S) is negligible compared to 1 in the non-resonant region. In this range $G_M(S)$ term dominates completely irrespective of $G_M(S)$ is minimum or non-minimum in nature. The phase angle will also be that of $G_M(S)$ which has also been verified by actual calculation.

But in the resonant region H(S) must reduce $G_1(S)$ to the form

$$G_1 (S) = \frac{G_M(S)}{1 + G_M(S)\ H(S)}$$

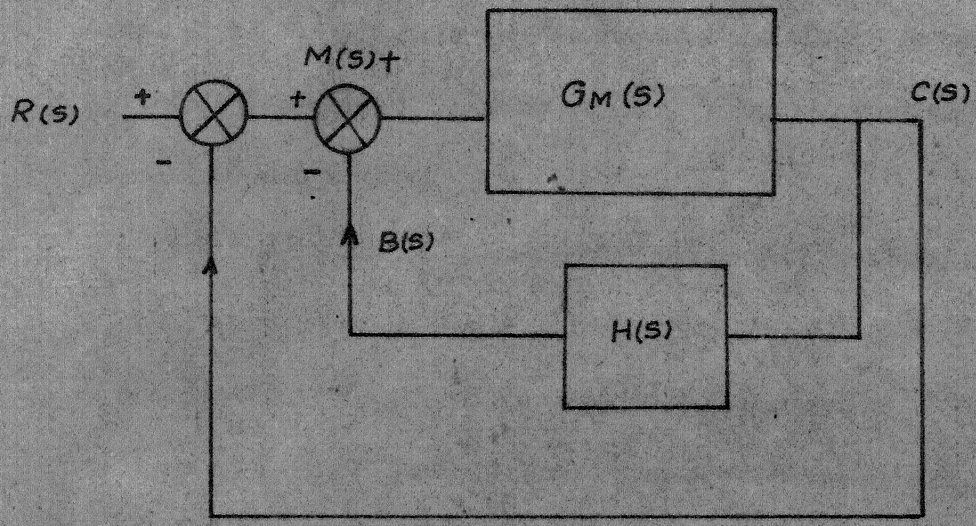
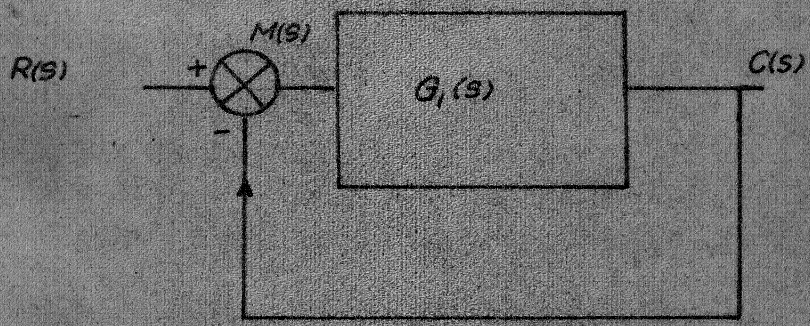


FIG. 3·1 FEED BACK COMPENSATION.



$G_M(S)$: OPEN LOOP TRANSFER FUNCTION AT MODIFIED GAIN.

$G_1(S)$ EQUIVALENT OPEN LOOP TRANSFER FUNCTION AFTER-COMPENSATION.

H (S) TRANSFER FUNCTION OF FEEDBACK COMPENSATOR

FIG 3·1

FEED BACK COMPENSATION

Here no approximation for $G_1(S)$ can be drawn as $G_M(S)\ H(S)$ will be close to 1. As a result $G_1(S)$ must be calculated from the exact formulae in this region. The usual asymptote approximation cannot be applied to this region.

A realisable H(S) transfer function which makes the $G_M(S)\ H(S)$ plot sufficiently negative in the non-resonance region in the Bode plot (To ensure $1 \gg G_M(S)\ H(S)$) can be obtained. But it must simultaneously satisfy the second constraint of attenuating the plot in the resonance region. So it is clear that the process of obtaining H(S) is not unique and involves trial and error process. The optimum values of the constants of H(S) are that which gives the maximum attenuation to the resonance phenomena without altering the plot in the other regions.

COMPENSATOR DESIGN

The open loop transfer function of the given system when the gain has been increased five times is approximately

$$G_M(S) \;\; \doteq \;\; \frac{120\ (1 + .08S)\ (1 - .0177S)}{S(1 + .418S)\left(\frac{S^2}{13,376} + \frac{48.4S}{13,376} + 1\right)}$$

From the plot shown in Fig. 2.4. The phase margin and gain margin at $K_v = 120$ are

Phase margin of the system = 34°

Gain margin of the system = 8 db

A general H(S) which makes $G_M(S)\,H(S) \ll 1$ in the non resonance region is of the form

$$H(S) = \frac{K_h\ S^2}{(1 + S\tau)}$$

The actual $G_M(S)\,H(S)$ curve depends upon the magnitude of K_h and τ. By choosing a very low value of K_h, $G_M(S)\,H(S)$ can be made negligible in comparison to 1. The value of τ was varied over a wide range and $G_1(S)$ was actually calculated in the resonance region for each value of τ. The value of τ that gave the desirable attenuation to the overshoot without appreciably altering the plot in other regions was chosen to be the optimum value.

Suitable values of the constants are $\tau = .00278$ and $K_h = .0001$. The magnitude and phase shift of $G_1(S)$ have been plotted in Fig. 3.2. The compensator transfer function H(S) has been plotted in Fig. 3.3.

The compensator design to realize the necessary modification is of the form

$$H(S) = \frac{.0001\ S^2}{(1 + .00278\ S)}$$

This compensator gives satisfactory attenuation to the resonant overshoot with negligible change in gain and phase margins.

The gain margin of the system after compensation = 8 db

The phase margin of the system after compensation = 36°

It is clear from Fig. 3.2 that the necessary modification has been obtained in the Bode plot through the compensator designed.

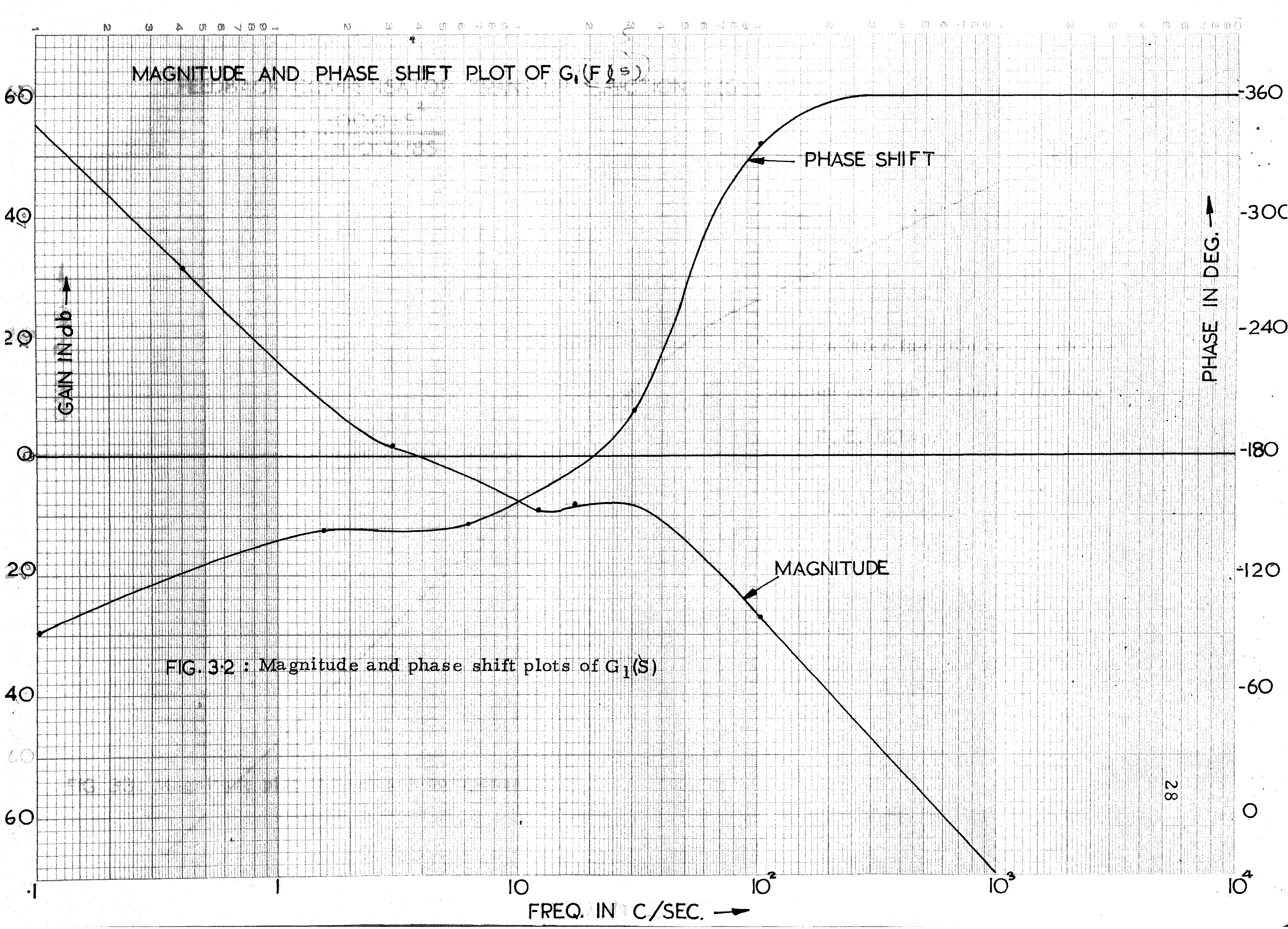


FIG. 3·2 : Magnitude and phase shift plots of $G_1(S)$

FEEDBACK COMPENSATOR TRANSFER FUNCTION H(S)

$$H(S) = \frac{\cdot 0001 S^2}{1 + \cdot 00278 S}$$

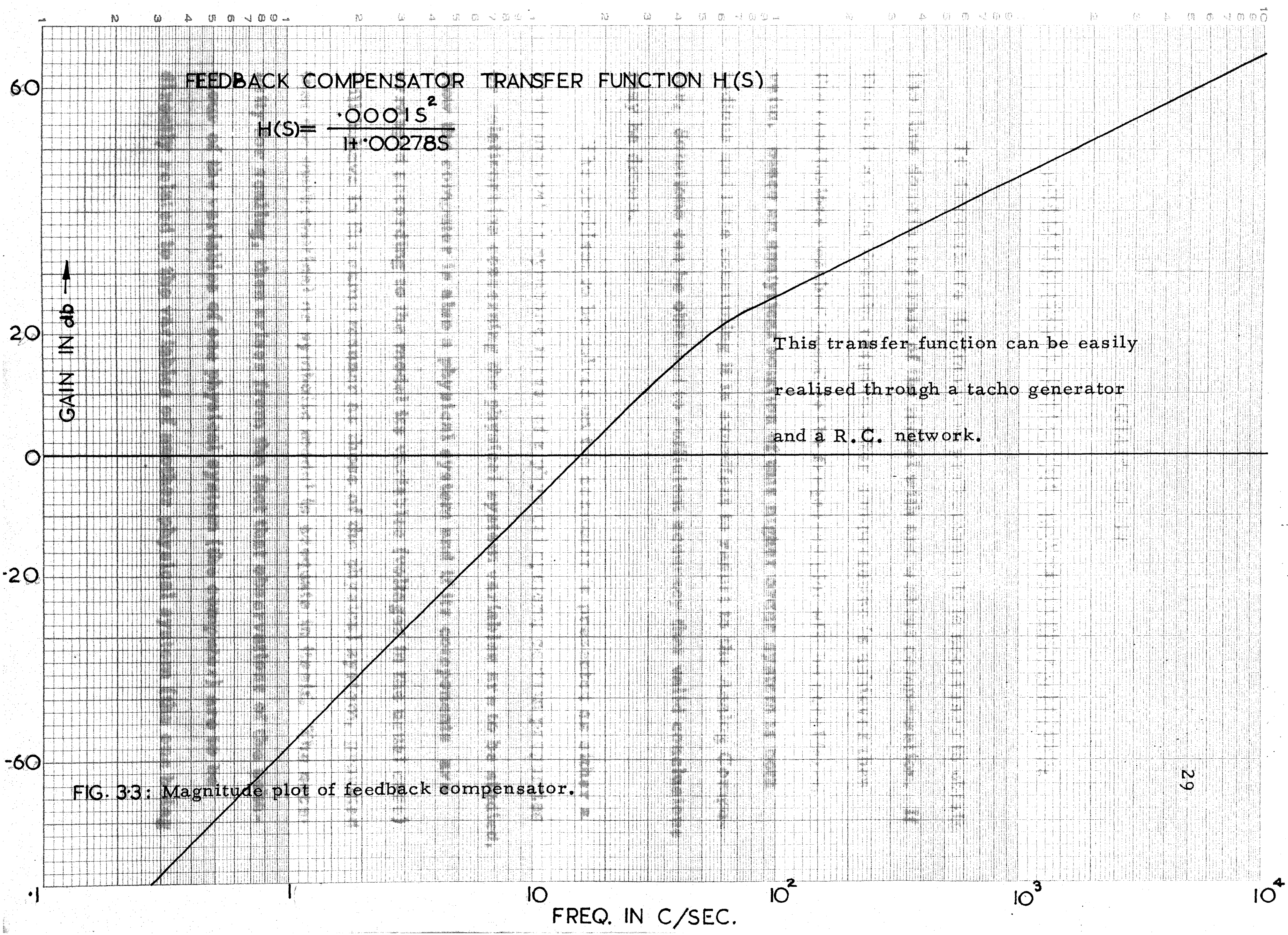


FIG. 3.3: Magnitude plot of feedback compensator.

CHAPTER IV

ANALOG SIMULATION OF THE MATHEMATICAL MODEL

To demonstrate the improved performance it is necessary to obtain the time domain response of the model with and without compensation. If the desired time domain response after compensation is achieved then the conclusions reached based on the frequency plot will be considered valid. Since an analytical solution of this higher order system is both tedious and time consuming it is expedient to resort to the Analog Computer. Solutions can be obtained to sufficient accuracy that valid conclusions may be drawn.

The problem to be solved on the computer is presented as either a mathematical or symbolic model of a physical system; certain properties or relationships concerning the physical system variables are to be studied. Now the computer is also a physical system and if its components are arranged according to the model its variables (voltages in the usual case) will behave in the same manner as those of the physical system, provided that the mathematical or symbolic model is adequate in itself. The necessity for scaling, then arises from the fact that observations of the behaviour of the variables of one physical system (the computer) are to be directly related to the variables of another physical system (the one being

studied). The variables in each system have very real limitations on their possible magnitude and rates of change with respect to time; hence the corresponding variables of the two systems must be related by magnitude and time scale factors. Since the given system is stable and the available computer has a range of 100 V (D.C.); magnitude scaling of the closed loop equation is not essential for block simulation for small input response. However time scaling is essential owing to wide difference in coefficient values in the equation.

METHODS OF SIMULATION

In general there are two basic approaches to setting up a computer to simulate a system (a) the differential equation method (b) block simulation method. In the first method the parameters of the system can be recognised in terms of some resistances or capacitances. However each parameter can be varied only by changing all the values of the capacitors, resistors and potentiometer settings associated with it. The inconvenience of this method is readily evident when it becomes necessary to study a system for a range of parameter variation. But in this method of simulation along with the output all the derivatives of the output are simultaneously available.

In block simulation the approach is to split up the given transfer function into x individual convenient blocks and generate them independently. In this method any physical parameter can be conveniently varied through a single component. The individual blocks can be tested and studied effectively

themselves. Differentiation can be avoided more readily by this approach.

The disadvantages of this method are

1. a greater number of amplifiers are required.

2. except for the first, derivatives of the output are not readily available in most systems.

In the differential equation method it is customary to rearrange the given equation in such a way that the highest order derivative is set equal to the rest of the terms in the equation. The next step is to assume that all the rest of the terms in the equation are available and that upon summation they produce the highest order derivative term. With the highest order derivative in hand successive integration yields the rest of the lower order terms. Multiplication of the lower order terms by the appropriate coefficients yields the term that were assumed available at the start of the process. The reference input signal is injected at the summation point and the block diagram is complete. If there are zeros in the transfer function then it will give rise to differentiation of input signal. This can be readily avoided by integrating successively both sides of the equation.

Consider the system to be simulated. This can be represented by a block diagram shown in Fig. 4.1.

$$C(S)/M(S) = \frac{K(1+\tau_1 S)(1-\tau_2 S)}{S(1+\tau_3 S)(aS^2+bS+1)}$$

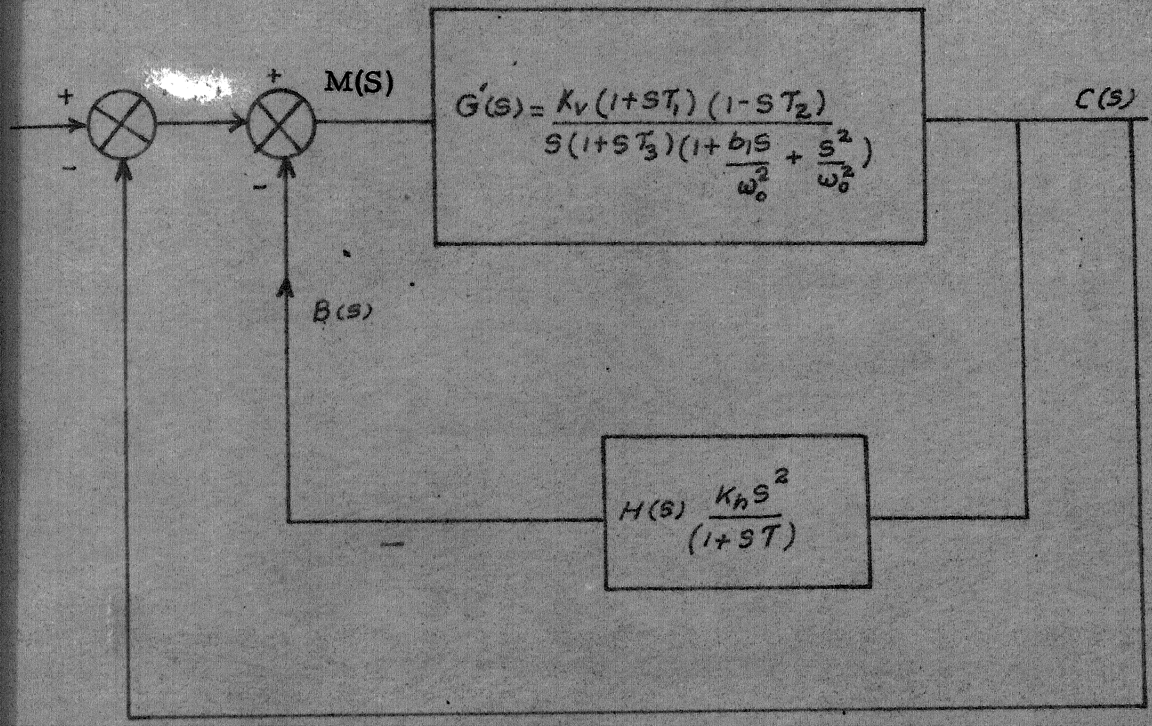


FIG. 4·1 BLOCK DIAGRAM OF THE SYSTEM TO BE SIMULATED.

$$K\,[\,1+(\tau_1-\tau_2)S-\tau_1\,\tau_2 S^2\,]\,M(S) = C(S)\,[\,(\tau_2 S^2+S)\,(aS^2+bS+1)\,]$$

$$K\,[\,1+(\tau_1-\tau_2)\,S-\tau_1\,\tau_2\,S^2\,]\,M(S) = C(S)\,[\,aS^3+bS^2+S+a\tau_2 S^4 + \tau_2\,bS^3+\tau_2\,S^2\,]$$

$$[\,K+K\,(\tau_1-\tau_2)\,S-K\,\tau_1\,\tau_2\,S^2]\,M(S) = C(S)\,[a\tau_2 S^4+(a+\tau_2 b)S^3 + (\tau_2+b)\,S^2+S\,]$$

Dividing throughout by S^2

$$M(S)\,[\,\frac{K}{S^2}+\frac{K\,(\tau_1-\tau_2)}{S}-K\,\tau_1\,\tau_2\,] = C(S)\,[\,a\tau_2\,S^2+(a+\tau_2 b)S + (\tau_2+b)+\frac{1}{S}\,]$$

for the feedback path

$$K_h\,S^2\,C(S) = (\tau S+1)\;B(S)$$

$$E(S) = R(S) - C(S)$$

$$M(S) = E(S) - B(S)$$

Rewriting the equation

$$a\tau_2\,S^2\,C(S) = \frac{K}{S^2}\;M(S) + \frac{K(\tau_1-\tau_2)}{S}\;M(S) - K\,\tau_1\;\tau_2\,M(S) -(a+\tau_2 b)\;S\,C(S) - (\tau_2+b)\,C(S) - \frac{C(S)}{S}$$

$$\tau\,S\;B(S) = K_h\,S^2\,C(S) - B(S)$$

The system block diagram for the above equation is shown in Fig. 4.2.

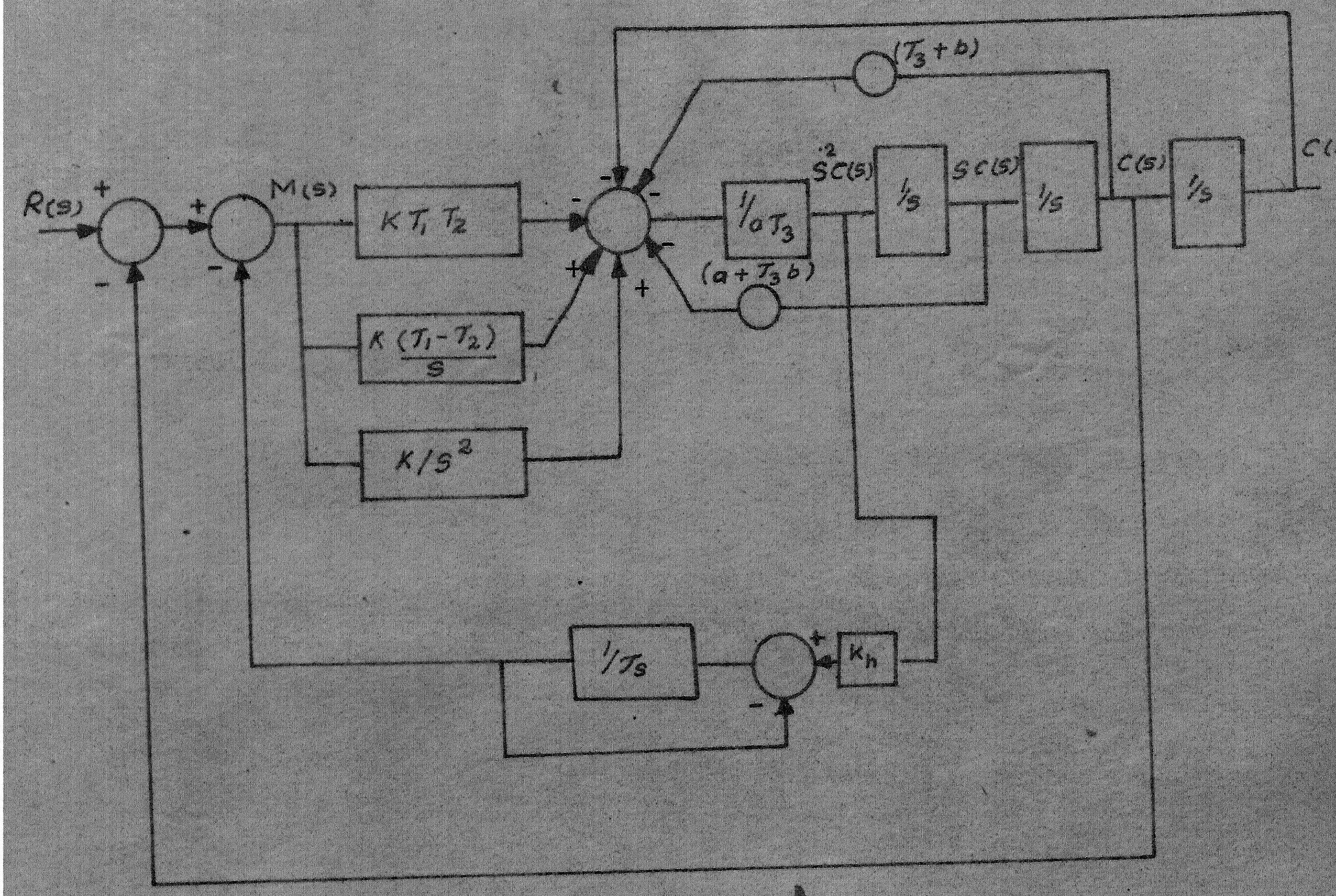


FIG. 4·2 SYSTEM BLOCK DIAGRAM.

The system formulated for the computer requires time scaling to assure that the variables do not exceed the permissible computer range. The modified system can be scaled for the computer as follows:

$$G_M(S) = \frac{120\ (1 + .08S)\ (1 - .0177S)}{S\ (1 + .418S)\ \left(\frac{S^2}{13,376} + \frac{48.4S}{13,376} + 1\right)}$$

By cross multiplying

$$.3 \times 10^{-4}\ S^4\ C(S) = 120\ M(S) + 7.44\ S\ M(S) - .171\ S^2\ M(S) - S\ C(S)$$
$$- .42\ S^2\ C(S) - .001575\ S^3\ C(S)$$

Since the coefficient of $S^4\ C(S)$ is very small compared to that of $S^3\ C(S)$ the time has to be slowed down on the computer. Integration on both sides essential to avoid differentiation of the input

Let $t = \tau/10$ i.e. $S = 10s$

The equation reduces to

$$.3\ s^2\ C(s) = [120\ \frac{M(s)}{s^2} + 74.4\ \frac{M(s)}{s} - 17.1\ M(s) - 10\frac{C(s)}{s} - 42\ C(s)$$
$$- 1.575\ s\ C(s)\ ]$$

$$.3\ s^2\ C(s) + 1.575\ s\ C(s) + 42\ C(s) + \frac{10\ C(s)}{s} = [\frac{120\ M(s)}{s^2} + \frac{74.4\ M(s)}{s}$$
$$- 17.1\ M(s)\ ]$$

$$s^2 C(s) + 5.25\, s\, C(s) + 140\, C(s) + 33.3\, \frac{C(s)}{s} = 400 \frac{M(s)}{s^2} + 248\, \frac{M(s)}{s} - 57\, M(s)$$

Now magnitude scaling the equation by the equal coefficient rule:

$$200 \left[\frac{s^2 C(s)}{200} \right] + 30 \times 5.25 \left[\frac{s\, C(s)}{30} \right] + 140 \left[C(s) \right] + 33.5 \times 5 \left[\frac{C(s)}{5s} \right]$$

$$= 400 \left[\frac{M(s)}{s^2} \right] + 248 \left[\frac{M(s)}{s} \right] - 57 \times 3 \left[\frac{M(s)}{3} \right]$$

$$\left[\frac{s^2 C(s)}{200} \right] + .79 \left[\frac{s\, C(s)}{30} \right] + .7 \left[C(s) \right] + .332 \left[\frac{C(s)}{2s} \right] = 2 \left[\frac{M(s)}{s^2} \right] + 1.24 \left[\frac{M(s)}{s} \right] - .285 \left[M(s) \right]$$

For the differential equation method a compensator close to that of the designed compensator has been taken to have an idea of the bandwidth of the feedback elements.

$$\text{Choosing } H(S) = \frac{.0005 S^2}{(1 + .005 S)}$$

Time scaling the equation H(S) reduces to

$$H(s) = \frac{.05\, s^2}{(1 + .05 s)}$$

The computer setup for the above mathematical model is shown in Fig. 4.3. The results of the Analog simulation are shown in page 39.

COMPUTER SET UP FOR CLOSED LOOP COMPENSATED SYSTEM

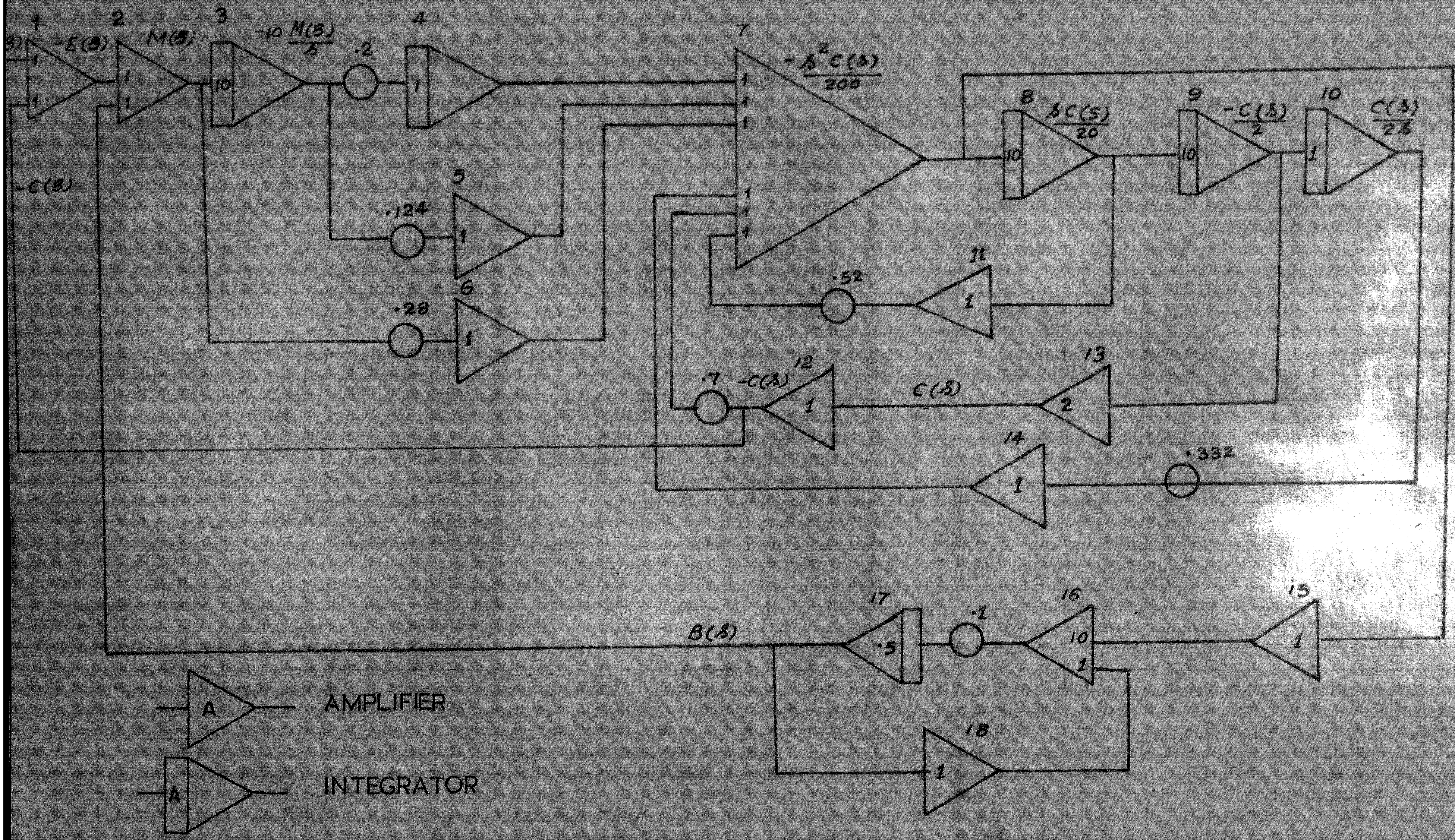


FIG. 4·3 DIFFERENTIAL EQUATION METHOD

BLOCK SIMULATION METHOD

In generating the simple operations of multiplication by a coefficient, integration and summation simple impedance structures for Zi (forward path impedance) Zf (feedback impedance) were used. If complex structures are considered then more complex transfer functions can be generated with a single operational amplifier. The block simulation method is different from the differential equation method of simulation in that one operational amplifier with complex impedances is used to generate transfer function wholly or part.

Consider the system transfer function

$$G_M(S) = \frac{120\,(1 + .08S)\,(1 - .0177S)}{S(1 + .418S)\left(\dfrac{S^2}{13,376} + \dfrac{48.4S}{13,376} + 1\right)}$$

H(S) has been taken to be the same as that of designed value.

$$H(S) = \frac{.0001\,S^2}{(1 + .00278S)}$$

Let $\dfrac{S^2}{13,376} = .1\,s^2$

$$S^2 = 1,337.6\,s^2 ; \quad S = 36.6\,s$$

Substituting for S

$$G(s)_M = \frac{120\,[1 + .08 \times 36.6s]\,[1 - .0177 \times 36.6\,s]}{36.6s\,[1 + .417 \times 36.6s]\,[.1s^2 + .1337s + 1]}$$

$$G(s)_M = \frac{3.28\,(1+2.93s)\,(1-.648s)}{s(1+15.25s)\,(.1s^2+.1337s+1)}$$

$$H(s) = \frac{.1\,s^2}{(1+.1s)}$$

$G(s)_M$ can be split into convenient blocks shown in Fig. 4.4(a). The problem can be simplified by coupling H(s) ahead of the last block as shown in Fig. 4.4(b). Accordingly one order of s has been reduced in the feedback loop block by this procedure. This reduces the difficulty in obtaining a block of above transfer function.

Finally the transfer function of the different blocks to be generated reduces to the form shown in Fig. 4.4(c).

The individual blocks can be obtained as follows:

Block A

$$\frac{e_o}{e_1} = \frac{1}{.1s^2+.1337s+1}$$

$$.1\,\ddot{e}_o = e_1 - .1337\,\dot{e}_o - e_o$$

$$\ddot{e}_o = 10\,e_1 - 1.337\,\dot{e}_o - 10\,e_o$$

Fig. 4.5(a) shows the computer set up for the above transfer function.

Block B

$$\frac{e_o}{e_1} = \frac{3.28\,(1-.648s)}{1+15.25\,s}$$

Consider the amplifier set up shown in Fig. 4.5(b)

$$e_o = -k\,e_x - e_y$$

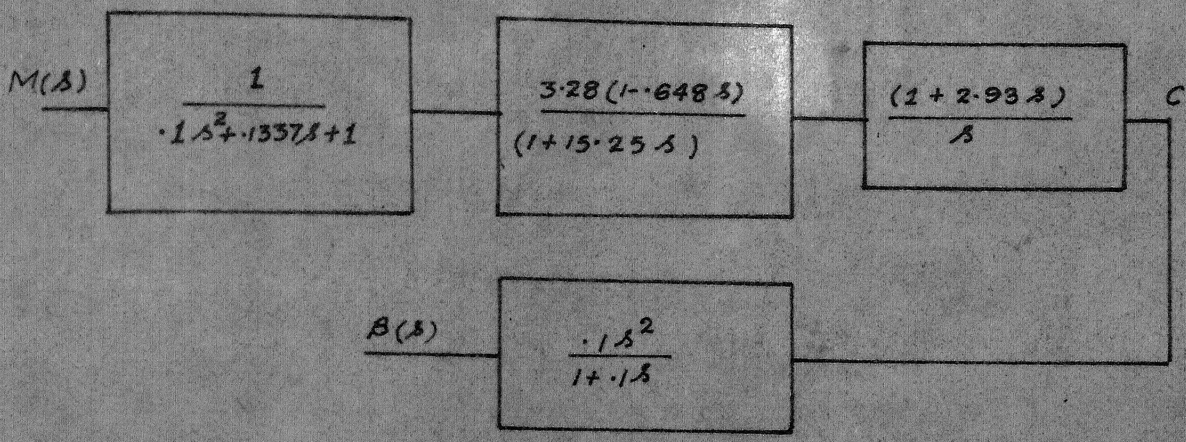


FIG. 4·4 a

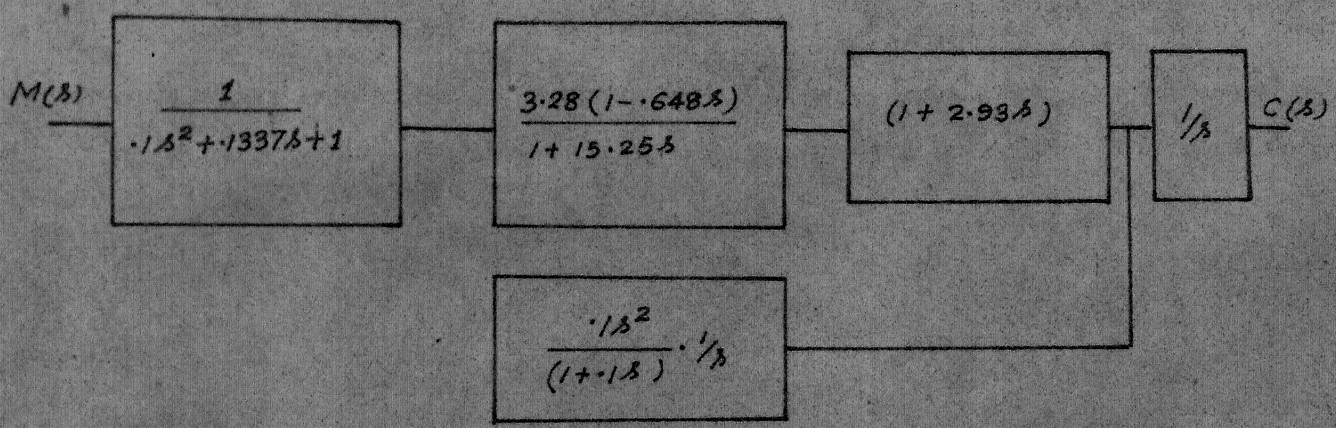


FIG. 4·4 b

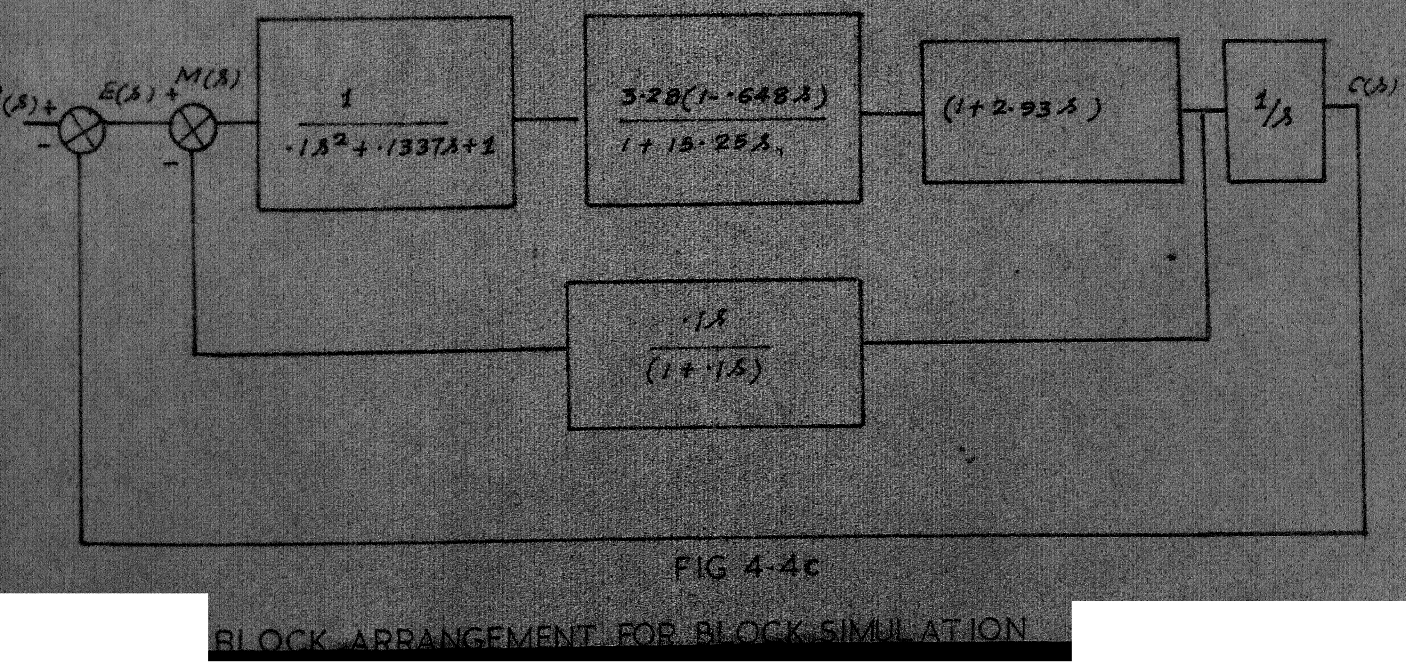


FIG 4·4c

BLOCK ARRANGEMENT FOR BLOCK SIMULATION

BLOCK A

$$e_o/e_i = \frac{1}{\cdot 1 s^2 + \cdot 1337 s + 1}$$

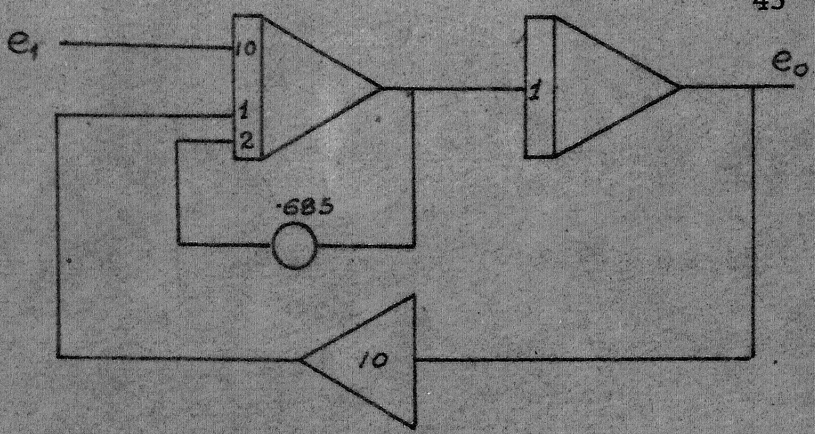


FIG 4·5a

BLOCK B

$$e_o/e_1 = -\frac{[1-(\frac{k}{a})s]}{[1+(\frac{1}{a})s]}$$

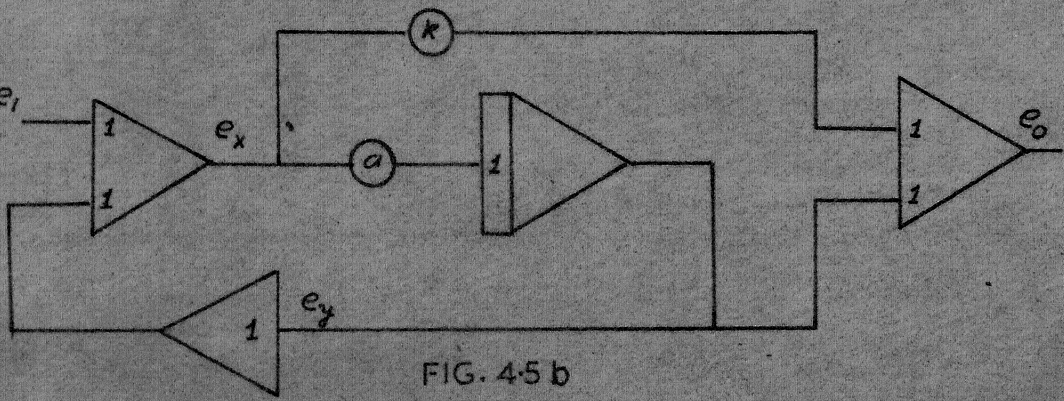


FIG. 4·5 b

BLOCK B

$$e_o/e_1 = \frac{3\cdot 28\,(1-\cdot 648 s)}{(1+15\cdot 25 s)}$$

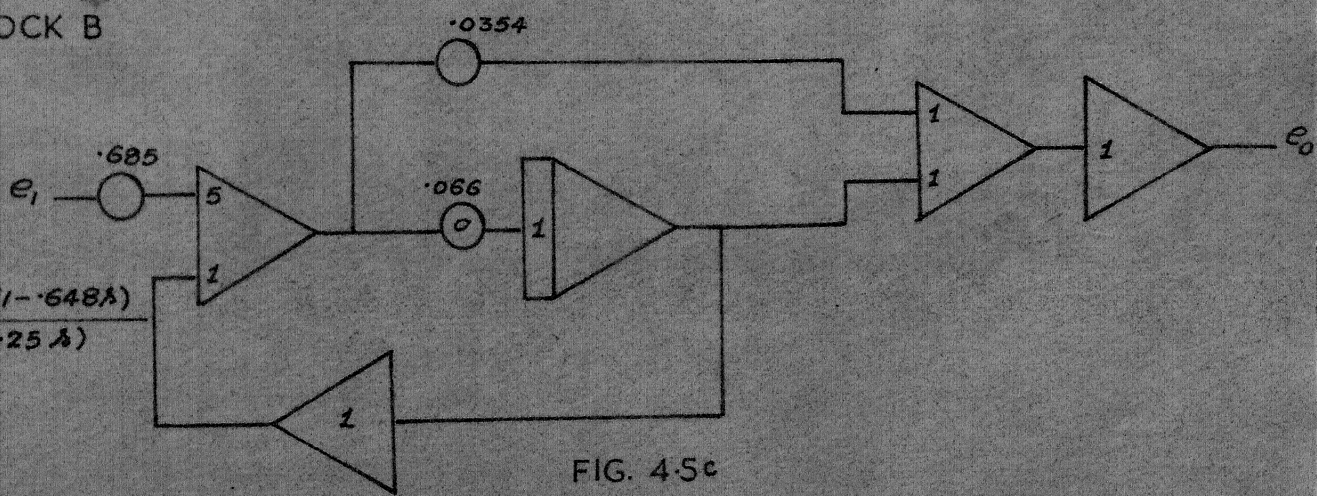


FIG. 4·5c

COMPUTER BLOCK SET UP

$$e_y = -\frac{a\, e_x}{s}$$

$$e_x = -e_1 + e_y$$

$$e_o = k\, e_1 - k\, e_y - e_y$$

$$e_y = -a/s \;\; (-e_1 + e_y)$$

$$e_y\,(s + a) = a\, e_1$$

$$e_o = k\, e_1 - (k + 1)\left[\frac{a\, e_1}{(s+a)}\right]$$

$$(s + a)\, e_o = e_1\,[\; ks + ka - ka - a \;]$$

$$\frac{e_o}{e_1} = -\left[\frac{a - ks}{a + s}\right]$$

$$\frac{e_o}{e_1} = -\left[\frac{(1 - (k/a)\, s)}{(1 + (1/a)\, s)}\right]$$

Here in the problem

$\frac{1}{a} = 15.25$ therefore $a = .066$

$\frac{k}{a} = .648\,;$ $k = .0425$

The gain of 3.28 can be picked up in the amplifier on the output or at the input. The computer set up for the block is shown in Fig. 4.5(c)

Block C

The simple zero (1 + 2.93s) can be obtained from a computer set up using single operational amplifier with particular transfer impedances connected to it.

Where 4 pole impedances are involved the short circuit transfer impedances are significant in obtaining a particular transfer function.

Consider the network shown in Fig. 4.6(a):

Let Z be the short circuit transfer impedance

$$Z = \frac{e_i}{i_o} \; ; \quad i_i = i_o + i_c$$

$$\frac{e_i - e_a}{R} = \frac{e_a}{R} + e_a\, c\, s$$

$$\frac{e_i}{R} = \frac{e_a}{R}\,(2 + R\,c\,s)$$

$$\frac{e_i}{R} = 2\,i_o\left(\frac{R\,c\,s}{2} + 1\right)$$

$$Z = \frac{e_i}{i_o} = 2\,R\left(\frac{R\,c\,s}{2} + 1\right)$$

This network can be used either as the input impedance (Z_i) or feedback impedance (Z_f) depending upon the desired transfer function. The required transfer function can be obtained by using the above network as feed back impedance as shown in Fig. 4.6 (b)

$$\frac{e_o}{e_i}(s) = \frac{Z_f(s)}{Z_i(s)} = -\frac{2R}{R_i}\left(\frac{R\,c}{2}\,s + 1\right)$$

Substituting the values $R_i = 12\ M.\Omega$ ax $R = 6\ M.\Omega$

$c = 1\,\mu.F$

$$\frac{e_o}{e_i}(s) = -(1 + 2.93\, s)$$

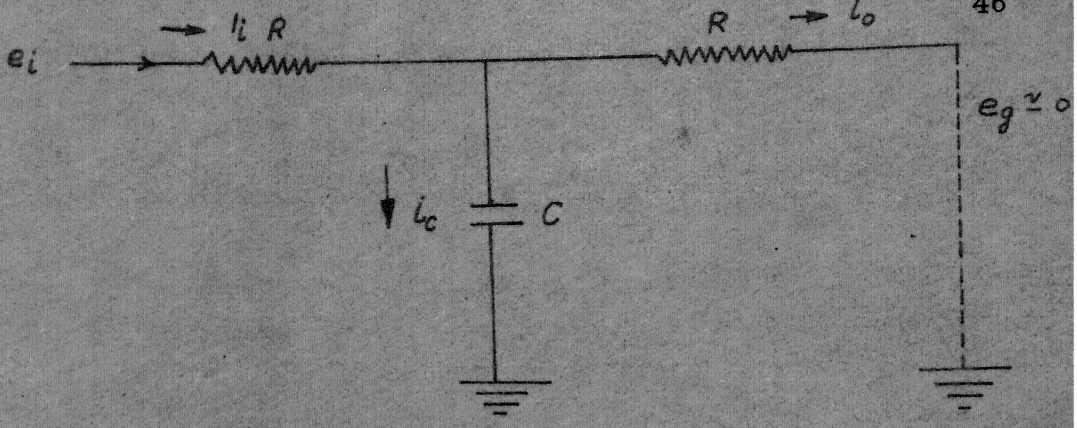
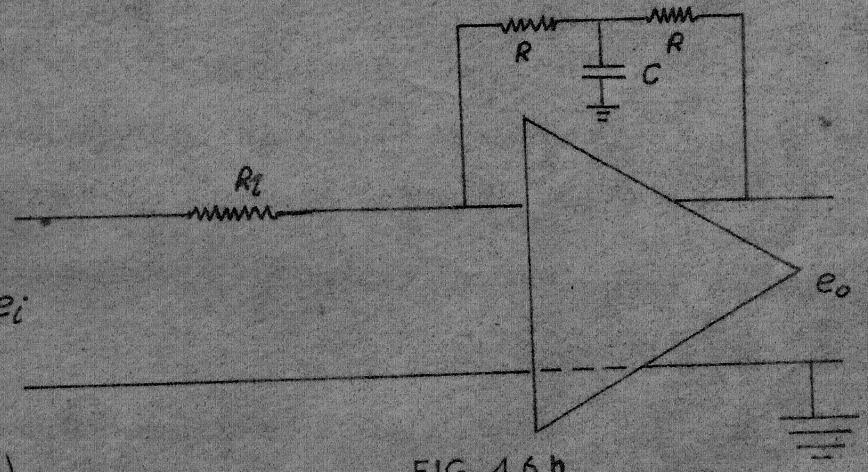
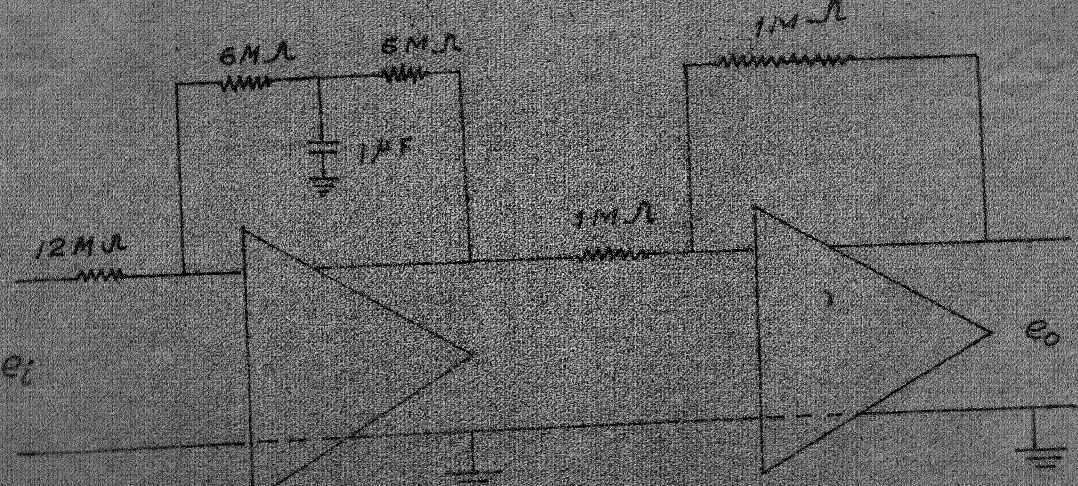


FIG. 4·6a



$$\frac{e_o}{e_i}(s) = -\frac{2R}{R_i}\left(\frac{RC}{2}s+1\right)$$

FIG. 4 6 b



BLOCK C

FIG. 4·6c

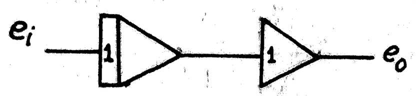
$$e_o/e_i \simeq (1 + 2.93s)$$

COMPUTER BLOCK SET UP

The computer set up for this block is shown in Fig. 4.6(c).

Block D

This can be obtained from a pure integrator and a unity gain amplifier.



$$\frac{e_o}{e_i}(s) = \frac{1}{s}$$

Fig. 4.7: Computer Set-up for Block D.

Block E

Consider a transfer function of the form

$$\frac{e_o}{e_i}(s) = \frac{.1\,s}{(1+.1s)}$$

This can be obtained by using a single operational amplifier similar to that of block C.

Consider the network shown in Fig. 4.7(a). The short circuit transfer impedance (Z) is of the form

$$Z = \frac{e_i}{i_o} = \frac{(\tau_o s + 1)}{B\,s} \qquad \text{where } B = c$$

$$\tau_o = R\,c$$

Applying this to the computer set up shown in Fig. 4.7(b).

$$\frac{e_o}{e_i}(s) = -\frac{R'\,c\,s}{(\tau_o s + 1)}$$

The required transfer function can be obtained by setting the values as

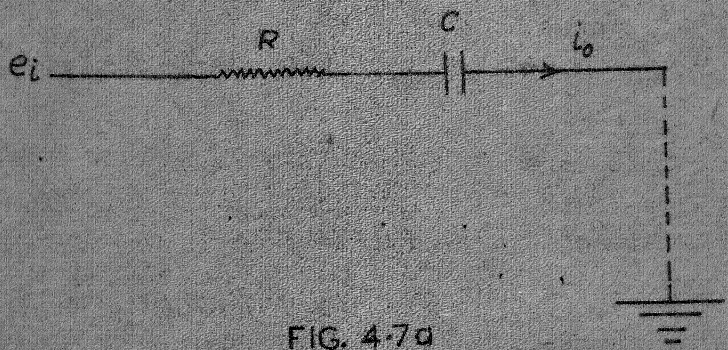


FIG. 4·7a

$$\frac{e_o}{e_i}(s) = \frac{-R'Cs}{(RCs+1)}$$

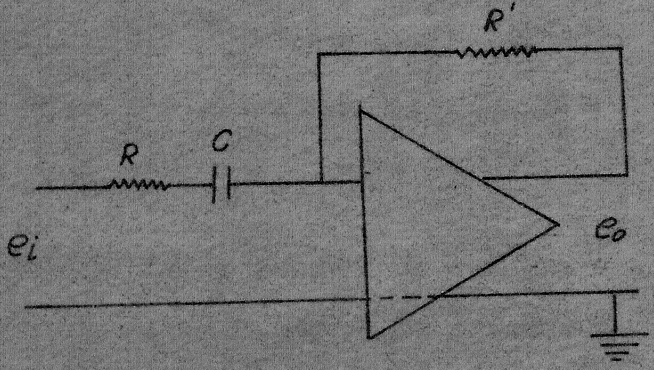


FIG. 4·7b

$$\frac{e_o}{e_i}(s) = \frac{\cdot 1 s}{(1+\cdot 1 s)}$$

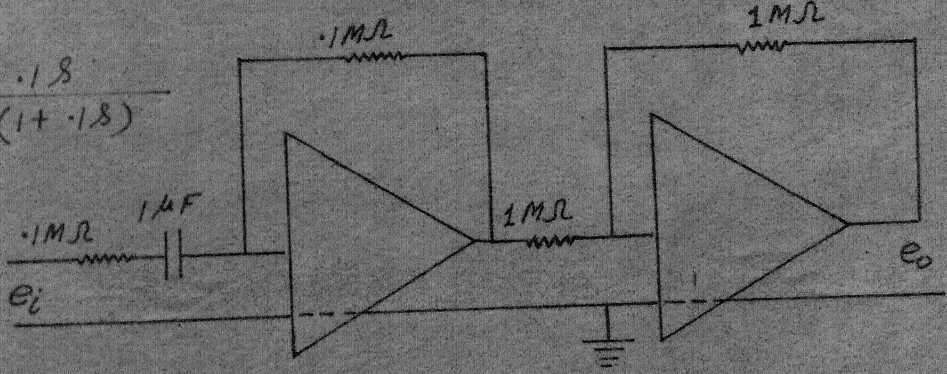


FIG. 4·7c

BLOCK E

COMPUTER BLOCK SET UP

follows:

$R' = .1 \, M.\Omega \; ; \; c = 1 \, \mu.F \; ; \; R = .1 \, M.\Omega$

The computer set up for the block is shown in Fig. 4.7 (c).

The computer set up for the entire system accomodating all the individual blocks is shown in Fig. 4.8. The results of Analog simulations (Block Simulation Method) are shown in page 51.

The following conclusions can be drawn from the results obtained by both differential and block simulation methods:

1. The system response is sluggish at low open loop gain (K_v) which is the case with the actual system.

2. An increase in gain makes the system more oscillatory.

3. With the compensator designed in circuit there is appreciable improvement in the settling time without exceeding the permissible overshoot.

The Analog computer simulation response gives a check both on the mathematical model obtained and the theoretical prediction made for the compensator design.

COMPUTER SET UP FOR CLOSED LOOP COMPENSATED SYSTEM

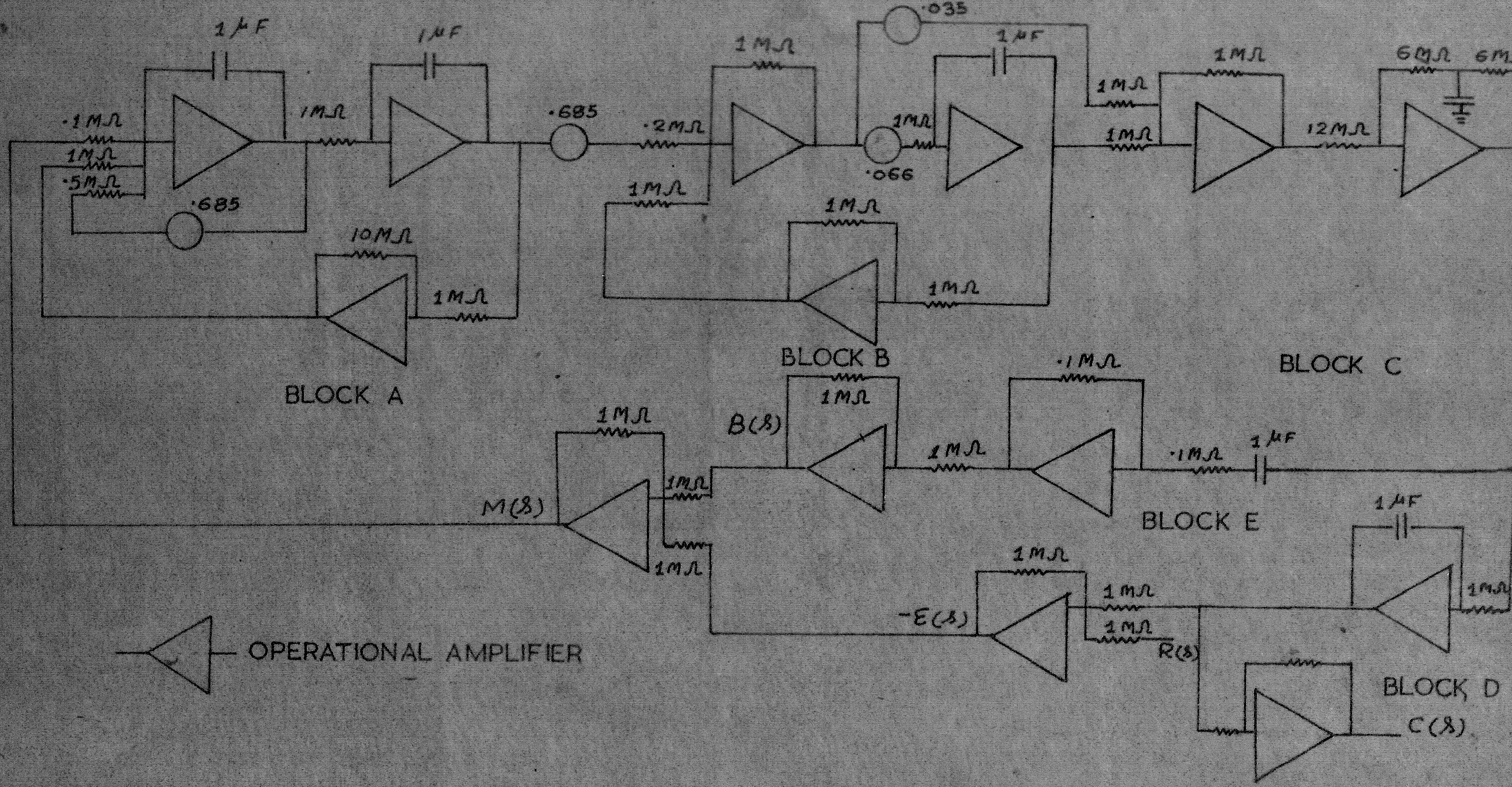
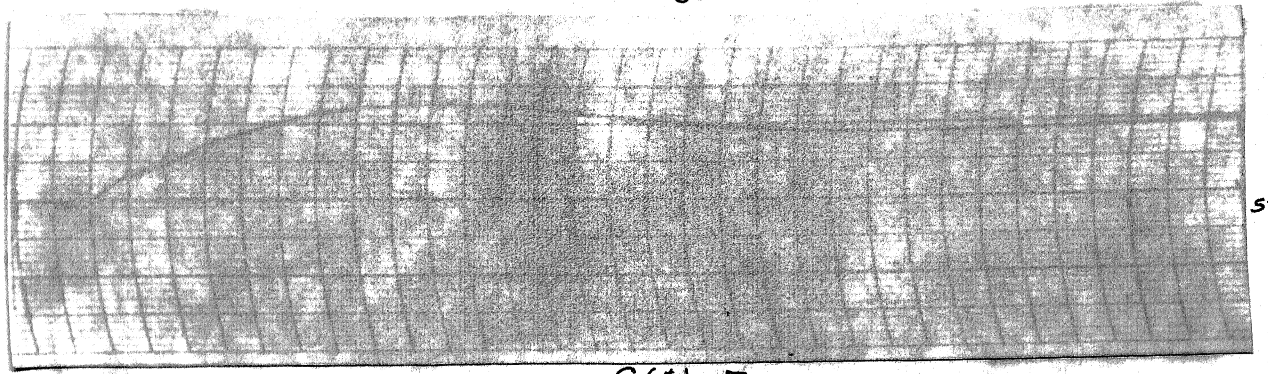


FIG. 4·8 BLOCK SIMULATION METHOD

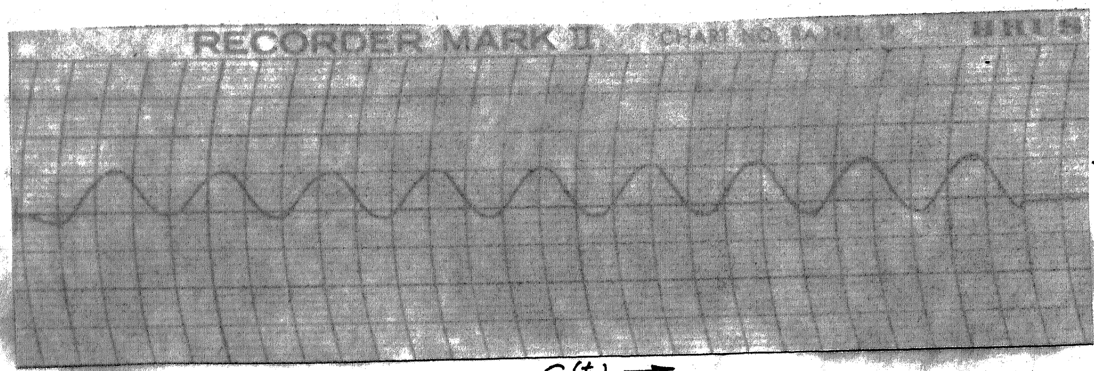
INPUT 5 V SCALE $\frac{1}{36.6}$ sec. = 5mm



s = .5V/Div

C(t) →

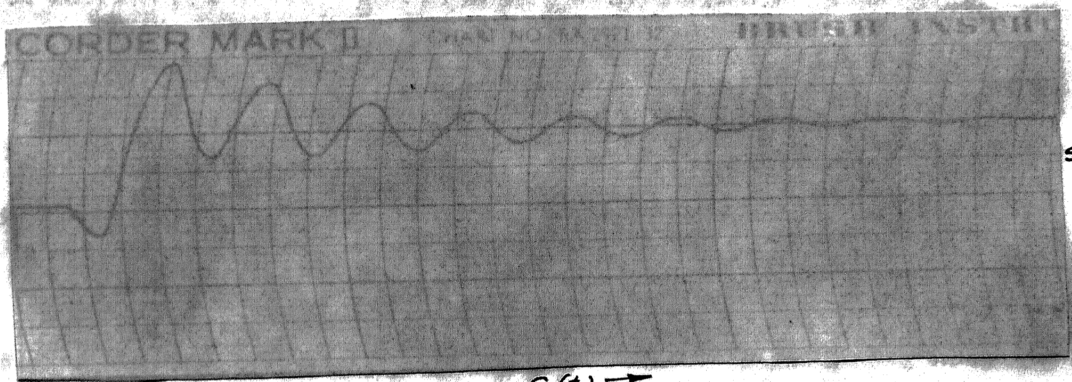
$Kv = 36$, $H(S)$ not in circuit



s = 2V/Div

C(t) →

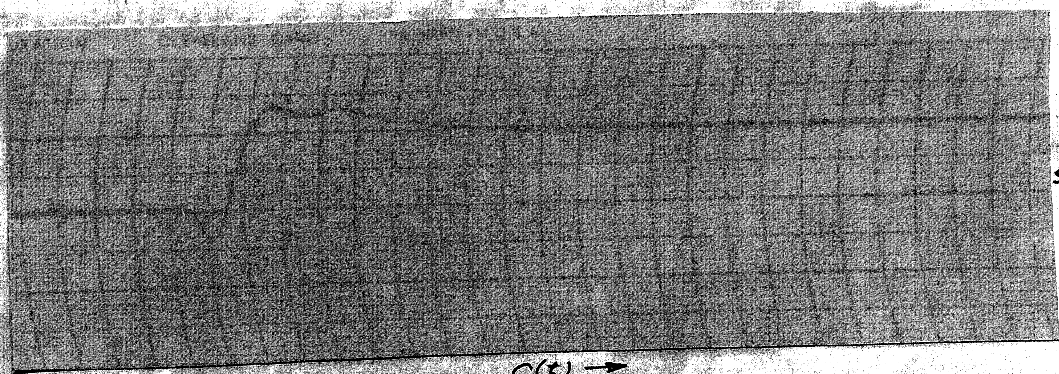
$Kv = 140$, $H(S)$ not in circuit



s = .5V/Div.

C(t) →

$Kv = 120$, $H(S)$ not in circuit



s = .5V/Di

C(t) →

$Kv = 120$, $H(S) = \dfrac{.0001S^2}{(1 + .00278S)}$ in circuit

OUTPUT RESPONSE C(t) [BY BLOCK SIMULATION METHOD]

CHAPTER V

DIGITAL VERIFICATION OF THE ANALOG MODEL

The analog computer is essentially a continuous device whereas the digital computer operates with discrete numbers. The danger of putting too much faith in the results of a computation is less with analog computation than it is with digital. In the digital computation the response can be obtained only at the discrete intervals. If the intervals are not carefully chosen nothing can be said about the solution between the two intervals in certain problems. But in analog computation the variables are continuous quantities rather than discrete numbers.

The analog computer is a less precise machine and is restricted to problems either where the answer is not required to a high degree of precision or the more common case where the input data are actually not known to any better precision. The accuracy of the result obtained in analog computation depends on:

1. The Measuring instruments.
2. The accuracy of operation of the computer (non linearities, noise and drift are primary causes of uncertainity).
3. The nature of the input data and the way in which it is entered in the machine.

4. The degree of confirmity of the mathematical model to the physical system.

The analog simulation and then the digital verification of the analog model together gives a thorough check on the system response.

For the given system a digital solution of the analog model was obtained through a computer programme on the IBM 1620. By the use of the programme the complete analog model can be constructed digitally and only the information about the nature of the blocks (amplifier or integrator), initial conditions, and potentiometer settings have to be given through punched data cards shown in Tables 3 and 4. The accuracy of integration can be made very high by using a very short integration interval. The computer will print the output at regular required print intervals over a prescribed length of time. Though scaling is not essential it is convenient to time scale the equation by the same factor used in analog computer simulation for comparison.

The system equation is of the form

$$\frac{C(S)}{M(S)} = G_M(S) = \frac{120\,(1 + .08S)\,(1 - .0177S)}{S(1 + .418S)\left(\frac{S^2}{13,376} + \frac{48.4S}{13,376} + 1\right)}$$

time scaling the equation by a factor ten ($t = \tau/10$).

The equation will reduce to the form

$$s^2\,C(s) = 400\,\frac{M(s)}{s^2} + 248\,\frac{M(s)}{s} - 57\;M(s) - 5.25\;s\;C(s)$$
$$-140\;C(s) - 33.3\;\frac{C(s)}{s}$$

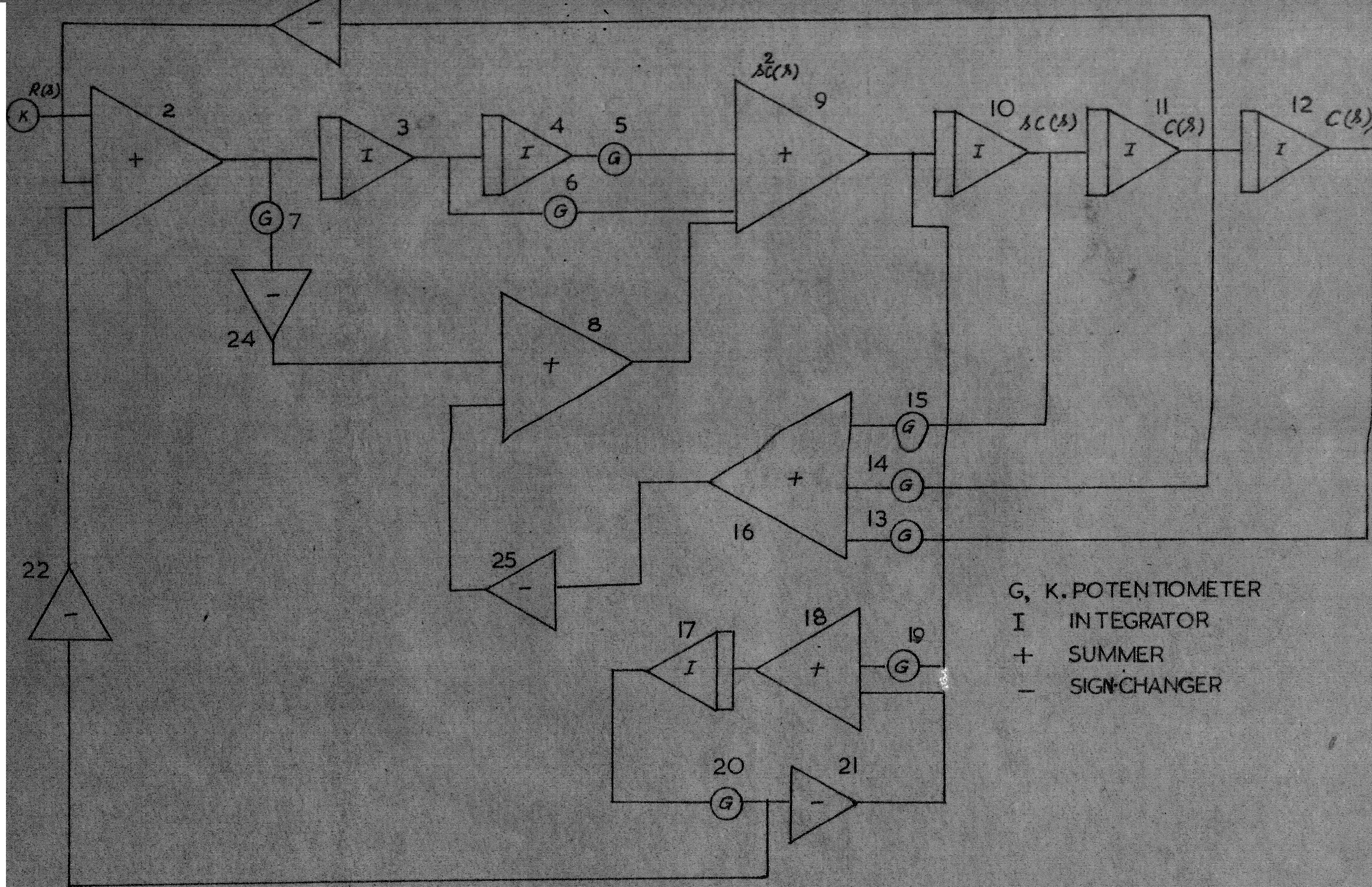


FIG. 5-1

ANALOG MODEL FOR DIGITAL SOLUTION

TABLE 3

COMPUTER DATA FOR ANALOG MODEL

Block (19-20)	Type (29-30)	Input 1 (39-40)	Input 2 (49-50)	Input 3 (59-60)
1	K			
2	+	1	23	22
3	I	2		
4	I	3		
5	G	4		
6	G	3		
7	G	2		
8	+	24	25	
9	+	5	6	8
10	I	9		
11	I	10		
12	I	11		
13	G	12		
14	G	11		
15	G	10		
16	+	13	14	15
17	I	18		
18	+	19	21	
19	G	9		
20	G	17		
21	-	20		
22	-	20		
23	-	11		
24	-	7		
25	-	16		

G, K : Represents Potentiometers

I : " Integrator

\+ : " Summer

\- : " Sign Changer

The analog model is shown in Fig. 5.1.

TABLE 4

INITIAL CONDITION PART

Block (19-20)	Part 1 (21-35)	Part 2 (36-50)	Part 3 (51-65)
1	10.0		
3	-		
4			
5	400.0		
6	248.0		
7	57.0		
10			
11			
12			
13	33.3		
14	140.0		
15	5.25		
17			
19	.01		
20	36.0		

Blank Card

OUTPUT REQUIRED

(1-2)	(3-4)	(5-6)
11	2	9

The feedback compensator designed is of the form

$$H(S) = \frac{.0001\,S^2}{(1 + .00278\,S)}$$

Scaling the equation ($t = \tau/10$). H (S) reduces to

$$H(s) = \frac{.01\,s^2}{(1 + .0278\,s)}$$

The analog system model for digital solution is shown in Fig. 5.1.

There is no sign change involved in amplifier or integrator output as the solution is being obtained digitally. The data also has to be given through cards punched in regular format as shown in Table 4. Any compensator value can be inserted by changing the data card involving feedback element. The programme has been stored in the tape in the Institute Computer Centre.

The following results were obtained by solving the Analog model on the digital computer with different H(S) in the feedback circuit.

(1)

With no Feedback Element H(S) in the Circuit

Loop gain K_v = 120

(a) Actual time (t)	(b) Computer Time (τ)	(c) Output 11 (C)	(d) Output 2 (M)	(e) Output 9 ($\ddot{C}$)
0.00	0.00	.000000E - 99	.100000E+02	-.570000E+03
.02	.20	-.375017E+01	.137501E+02	.439885E+03
.04	.40	.506801E+01	.493200E+01	.120060E+03

(a)	(b)	(c)	(d)	(e)
.06	.60	.177269E+02	-.772790E+01	-.496720E+03
.08	.80	.144374E+02	-.443740E+01	-.696500E+02
.10	1.00	.817180E+01	.182820E+01	.335560E+03
.12	1.20	.124036E+02	-.240360E+01	-.464900E+02
.14	1.40	.155275E+02	-.552750E+01	-.249360E+03
.16	1.60	.106736E+02	-.673600E+00	.891100E+02
.18	1.80	.837089E+01	.162920E+01	.170500E+03
.20	2.00	.116112E+02	-.161120E+01	-.987100E+02
.24	2.40	.896081E+01	.103920E+01	.999700E+02
.28	2.80	.110655E+02	-.106550E+01	-.847900E+02
.32	3.20	.904184E+01	.958200E+00	-.689800E+02
.36	3.60	.106913E+02	-.691300E+00	-.517000E+02
.40	4.00	.945519E+01	.552000E-01	.100400E+02
.44	4.40	.103605E+02	-.360500E+00	-.238400E+02
.48	4.80	.976720E+01	.232800E+00	.141000E+02
.52	5.20	.101324E+02	-.132400E+00	-.713000E+01
.56	5.60	.993949E+01	.606000E-01	.241000E+01
.60	6.00	.100154E+02	-.154000E-01	.460000E+00
.64	6.40	.100127E+02	-.127000E-01	-.202000E+01

C: System output

Input = 10.00

(2)

With Feedback Element $H(S) = \dfrac{.0005\,S^2}{(1+.005S)}$ in circuit.

Loop gain K_v = 120 Input = 10.00 C: System Output

Actual time (t)	Computer time (τ)	Output 11 (C)	Output 2 (M)	Output 9 ($\ddot{C}$)
0.00	0.00	.000000E-99	.100000E+02	-.570000E+03
.02	.20	-.396221E+01	.122118E+02	.562897E+03
.04	.40	.665844E+01	.260368E+01	-.939100E+02
.06	.60	.141110E+02	-.325468E+01	-.397530E+03
.08	.80	.116084E+02	-.201032E+01	.197730E+03
.10	1.00	.124919E+02	-.297119E+01	.120200E+02
.14	1.40	.116360E+02	-.179025E+01	.779500E+02
.18	1.80	.114246E+02	-.119278E+01	-.503300E+02
.22	2.20	.100581E+02	-.181663E+00	.144000E+02
.26	2.60	.996030E+01	.311030E-01	.883000E+01
.30	3.00	.100114E+02	.174956E-01	-.749000E+01

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(3)

With Designed compensator $H(S) = \dfrac{.0001S^2}{(1+.00278S)}$ in the feedback path

Look gain K_v = 120 Input = 10.00 C: System Output

Actual time (t) (a)	Computer time (τ) (b)	Output 11 (C) (c)	Output 2 (M) (d)	Output 9 ($\ddot{C}$) (e)
0.00	0.00	.000000E-99	.100000E+02	-.570000E+03

(a)	(b)	(c)	(d)	(e)
.02	.20	-.392099E+01	.688500E+01	.769154E+03
.04	.40	.793922E+01	.548310E+01	-.436220E+03
.06	.60	.112235E+02	-.137862E+01	.768700E+02
.08	.80	.135869E+02	-.288328E+01	-.994200E+02
.10	1.00	.137312E+02	-.353939E+01	.103000E+01
.14	1.40	.123637E+02	-.232789E+01	.153000E+01
.18	1.80	.109240E+02	-.953278E+00	.372000E+01
.22	2.20	.101810E+02	-.207991E+00	.269000E+01
.24	2.40	.100162E+02	-.416719E-01	.238000E+01

As expected the values of C(t) (System output) agrees closely with that obtained by Analog Computer simulation. It can also be seen from both the methods that at the modified gain $K_v = 120$ the designed compensator gives considerable improvement on the system performance.

RESULTS AND DISCUSSION

The following observations can be made on the time domain solution of the system mathematical model. The given system is extremely sluggish and takes 1.3 sec. to settle down. This is relatively large in comparison with the test time of the models in wind tunnel which is normally 3-4 secs. The peak overshoot is 25% (approx.) of the final value. When the loop gain has been increased to 120 (K_v) the system becomes more oscillatory in nature. The peak overshoot at this gain value is around 77% of the final value though the settling time reduces to .65 sec.

With the designed compensator $H(S) = \dfrac{.0001S^2}{(1 + .00278S)}$ in the beedback path the system performance measures can be tabulated as follows for comparison:

Performance criteria	Compensated system	Existing System
1. Settling time	.24 sec.	1.3 sec.
2. Loop gain (K_v)	120	22.7
3. Overshoot (in % of final value)	37%	25%
4. Rise time	.035 sec.	.2 sec.
5. t_p (time taken to reach first peak)	.1 sec.	.4 sec.

Initially the closed loop step response swings negative owing to a non-minimum zero present in the system. It is clear from the analog simulation response shown earlier that the maximum values to which it goes negative is a function of the loop gain K_v. It can also be observed that at the operating gain of 22.7 the effect of the non-minimum zero is negligible as far as the initial negative response is concerned. The effect of the relativbly large negative swing is not detrimental to the performance of the system as the aero models are always tested at the steady state conditions (when the settling chamber pressure is constant).

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